UBC Physics 102

Lecture 12

Rik Blok

Outline

- ▷ Straight wire
- ▷ Force between wires
- ▷ Ampere's law
- ▷ Solenoids and toroids
- ▷ End

Straight wire [Text: Sect. 28-1]

- **Discussion: Straight wire**

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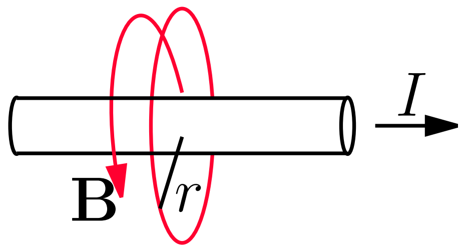
  - Magnetic field due to current in a long straight wire.



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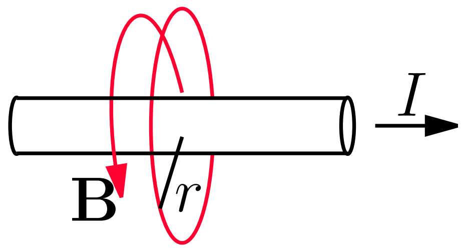

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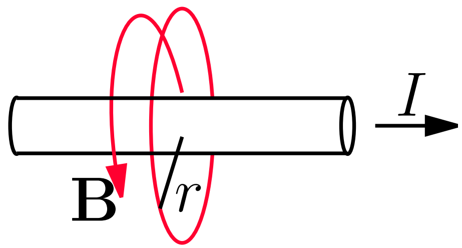

  - Stronger closer to wire, $B \propto \frac{1}{r}$, and with stronger current, $B \propto I$.
  - Will derive later that

$$\boxed{B = \frac{\mu_0}{2\pi}\frac{I}{r}.}$$

Force between wires [Text: Sect. 28-2]

- **Definition:** *Permeability of free space,* μ_0

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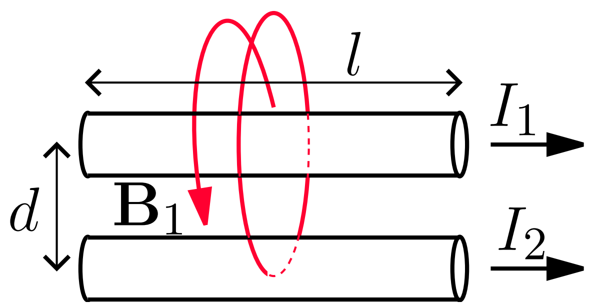
  - If wires also produce B-fields then 2 parallel wires will have force on each other.



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- **Interactive Quiz: PRS 12a**

Ampère's law [Text: Sect. 28-4]

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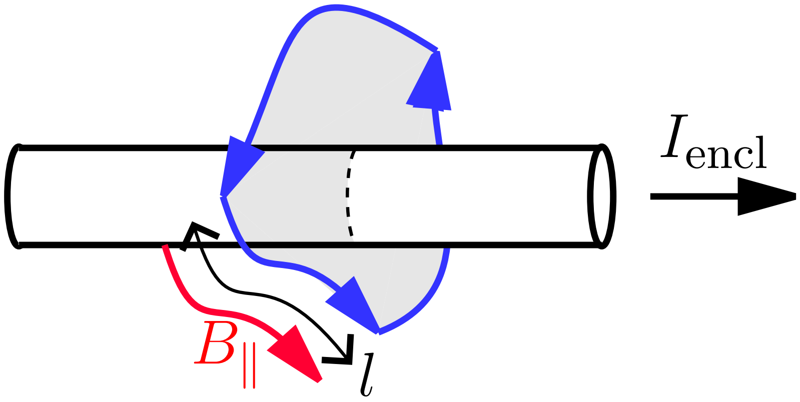

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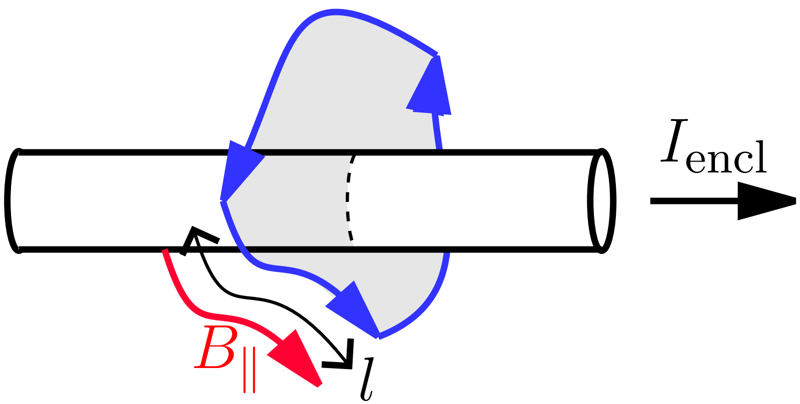

- If a current I_{encl} passes through a closed loop then

$$\boxed{\sum_{\text{segments}} B_{\parallel} l = \mu_0 I_{\text{encl}}.}$$

Ampère's law, contd

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  - Second of Maxwell’s 4 equations.

Ampère's law, contd

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  - You get to choose "Amperian" loop. Use symmetry.
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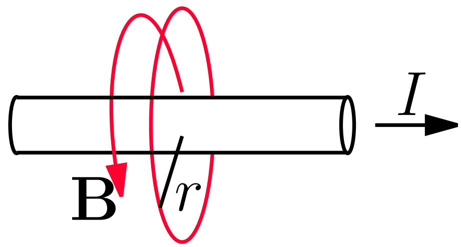
  - So we pick circular Amperian loop (1 continuous segment).

Ampère's law, contd

- **Derivation: Long, straight wire, contd**

Ampère's law, contd

- **Derivation: Long, straight wire, contd**
  - $B_{\|} = B$ everywhere on circle.



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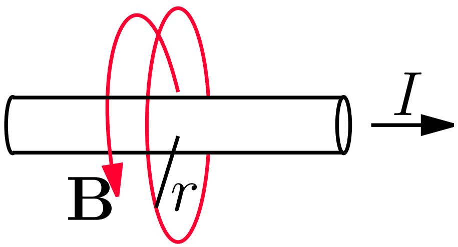

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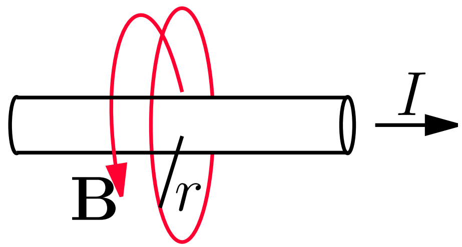

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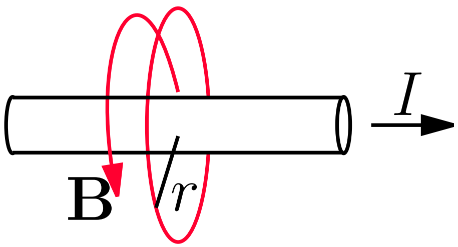

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$$\sum_{\text{segments}} B_{\parallel} l = \mu_0 I_{\mathrm{encl}}$$

$$B(2\pi r) = \mu_0 I.$$

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  - So we find

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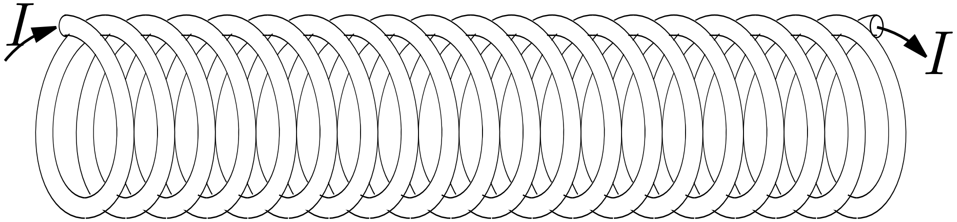

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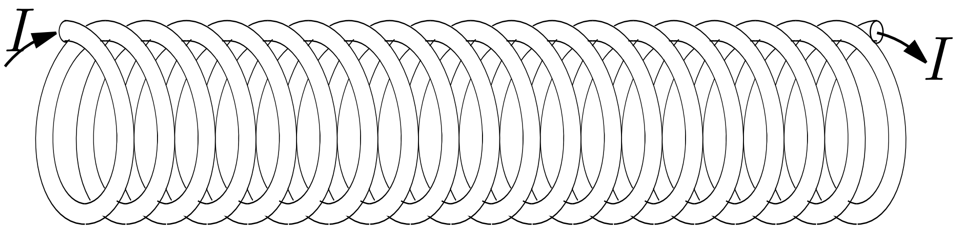

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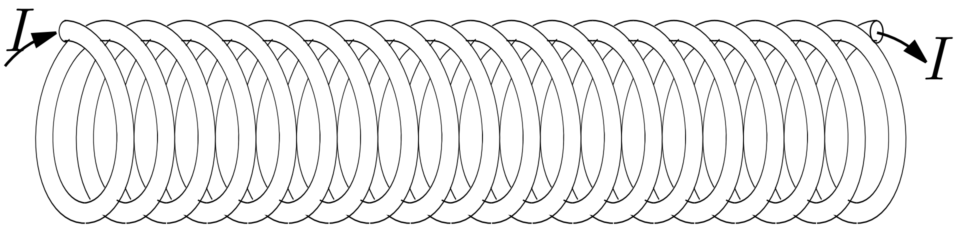
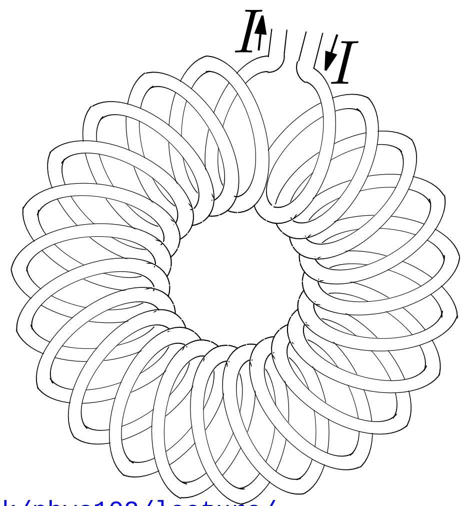


- **Definition:** *Toroid*
  - Solenoid bent into the shape of a donut (torus).



Solenoids and toroids, contd

- **Principle: Superposition**

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  - Like E-field, can find net B-field by adding up B's due to each wire.

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  - Can use Ampère's law to find B-field in/around toroid.
  - By symmetry loop should be circle of radius r.
  - 3 cases: (1) loop smaller than toroid, (2) loop inside toroid, (3) loop bigger than toroid.

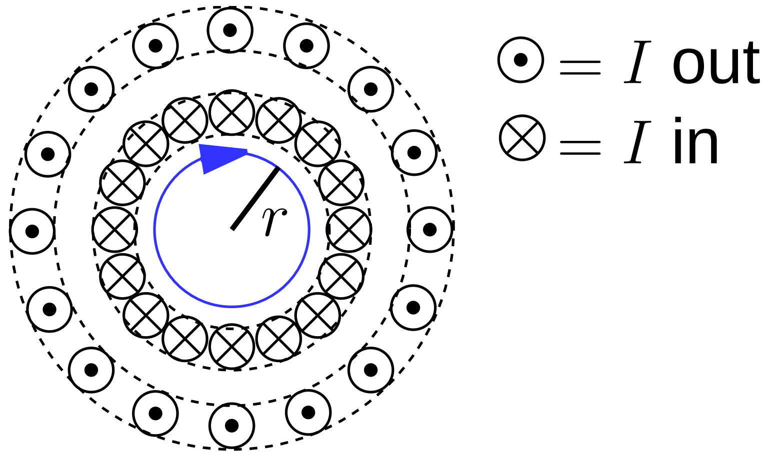
Solenoids and toroids, contd

- **Derivation: Toroid magnetic field, contd**

Solenoids and toroids, contd

Derivation: Toroid magnetic field, contd

Case 1: Cross-sectional view of toroid:



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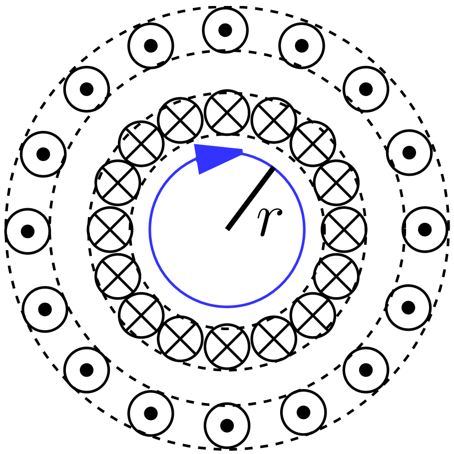

$\odot = I$ out
$\otimes = I$ in

- $I_{\text{encl}} = 0$ and $B = B_{\parallel}$ so for any r we find

$$B = 0.$$

Solenoids and toroids, contd

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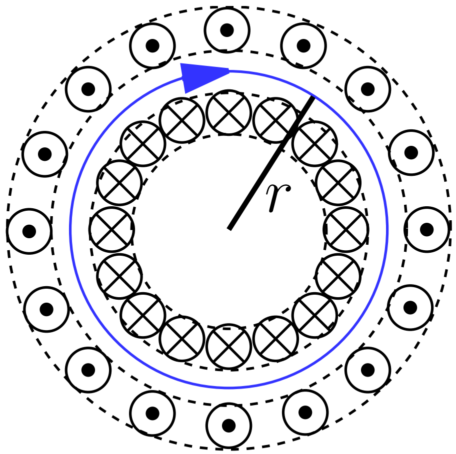
Case 2: Cross-section:



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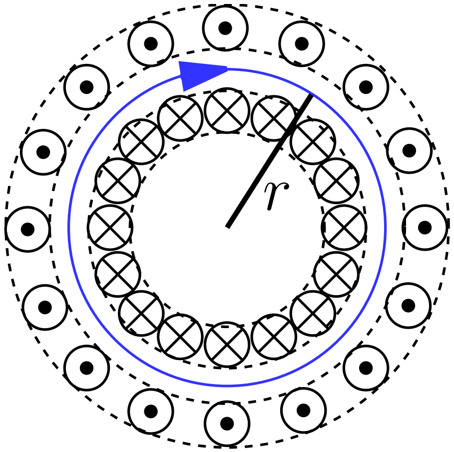

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- If there are N turns then $I_{\text{encl}} = NI$ so

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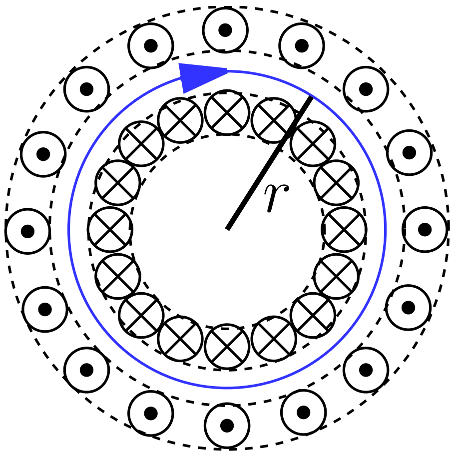

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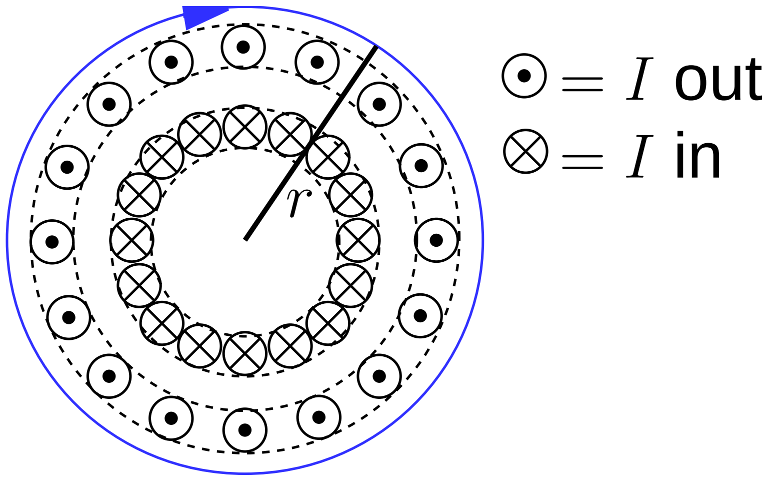

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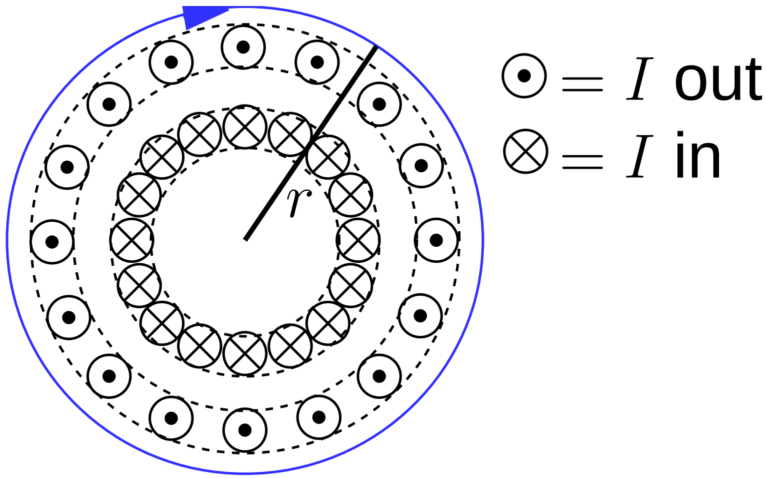

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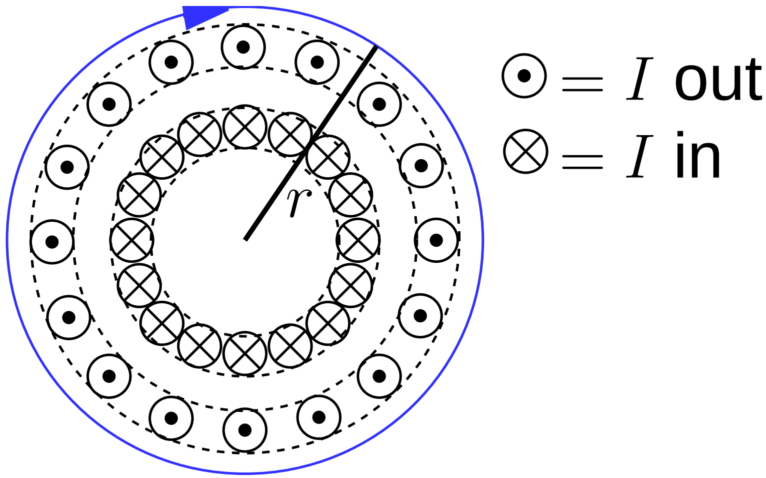

- Again, $I_{\text{encl}} = 0$ (they all cancel) so

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- So $B = 0$ everywhere outside toroid and $B = \mu_0 \frac{N}{l} I$ inside. $\square$

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- **Derivation: Solenoid magnetic field**

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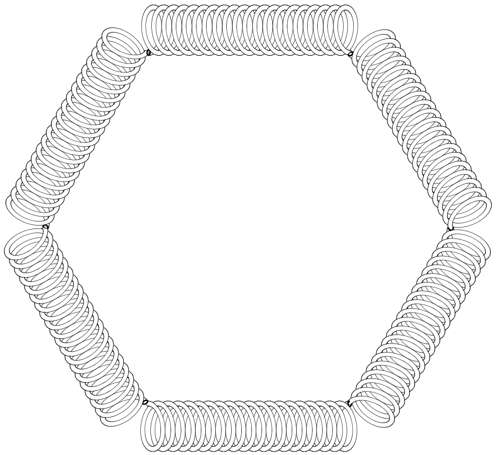
  - We can construct a toroid from many solenoids laid in a circle.



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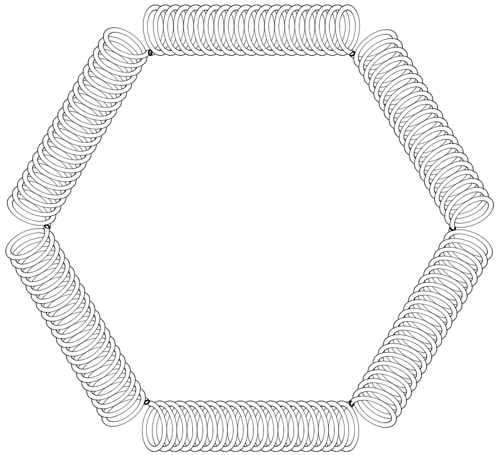

  - So each solenoid must have same field,

$$\boxed{B = \mu_0 \frac{N}{l} I.}$$

Solenoids and toroids, contd

- **Derivation: Solenoid magnetic field, contd**

Solenoids and toroids, contd

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  - Behaves like bar magnet (B comes out of N, goes into S end.)

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- **Derivation: Solenoid magnetic field, contd**
  - And $B = 0$ (roughly) outside solenoid.
  - $\frac{N}{l}$ is # turns per unit length, often written n (eg. $B = \mu_0 n I$).
  - Use RH field rule to determine direction.
- **Discussion: Solenoid**
  - B very uniform inside solenoid (far from ends).
  - B gets weaker and starts to spread near ends.
  - Behaves like bar magnet (B comes out of N, goes into S end.)
- **Interactive Quiz: PRS 12c**

End

- **Practice Problems:**
  - Ch. 28: Q. 1, 3, 5, 7, 9, 11, 21, 23.
  - Ch. 28: Pr. 1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23 ,25, 27, 47, 49, 55, 59, 61, 63.

End

- **Practice Problems:**
  - Ch. 28: Q. 1, 3, 5, 7, 9, 11, 21, 23.
  - Ch. 28: Pr. 1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23 ,25, 27, 47, 49, 55, 59, 61, 63.
- **Interactive Quiz: Feedback**

End

- **Practice Problems:**
  - Ch. 28: Q. 1, 3, 5, 7, 9, 11, 21, 23.
  - Ch. 28: Pr. 1, 3, 5, 7, 9, 11, 13, 15, 17, 21, 23 ,25, 27, 47, 49, 55, 59, 61, 63.
- **Interactive Quiz: Feedback**
- **Tutorial Question: tut12**