UBC Physics 102

Lecture 10

Rik Blok

Outline

- ▷ Emf
- ▷ Resistors in series and parallel
- ▷ Kirchhoff's rules
- ▷ RC Circuits
- ▷ End

Emf [Text: Sect. 26-1]

- **Definition:** *Emf*, $\mathscr{E}$

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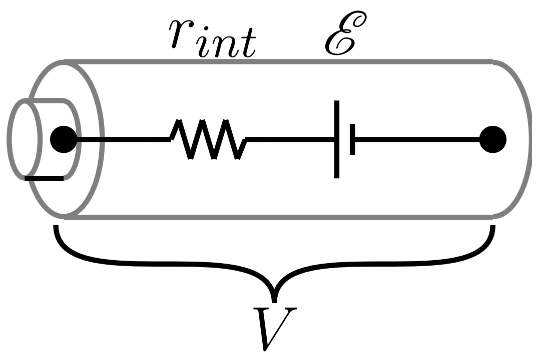
- **Discussion: Batteries**
  - Batteries have internal resistance, r_{int}.
  - Some voltage loss across r_{int}.



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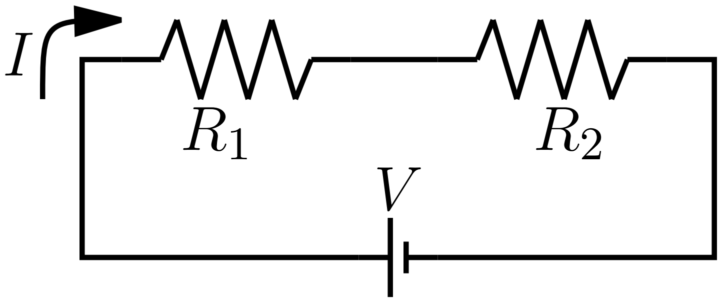
Emf, contd

- **Discussion: Batteries, contd**
  - When current I flowing net voltage across terminals is
    $$V = \mathscr{E} - r_{int} I.$$
  - $\mathscr{E}$ and r_{int} properties of battery (Lab 3).
  - In problems you may see $\mathscr{E}$ instead of V. Assume no internal resistance.

Resistors in series and parallel [Text: Sect. 26-2]

- **Derivation: Resistors in series**



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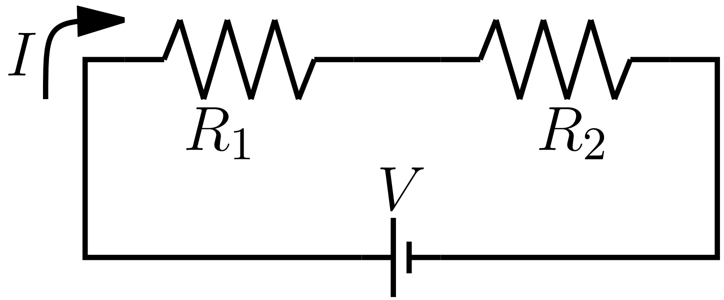

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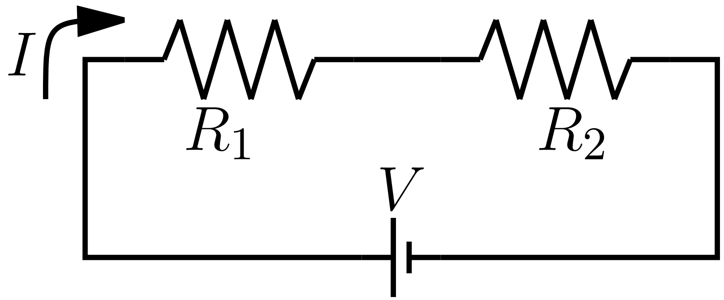

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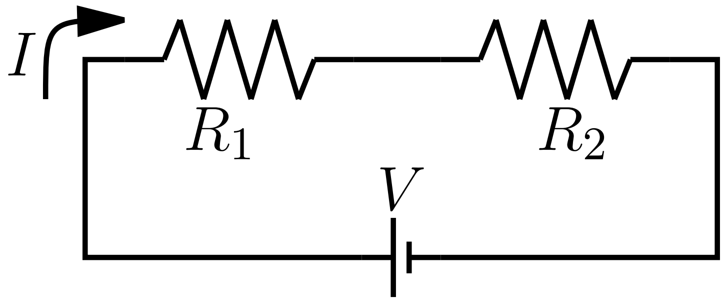

  - No branches so same current goes through both resistors.
  - Voltage drop across each resistor: $V_1 = IR_1$, $V_2 = IR_2$.
  - Voltage drop must equal voltage gain of battery,

$$
\begin{aligned}
V &= V_1 + V_2 \\
&= I(R_1 + R_2).
\end{aligned}
$$

Resistors in series and parallel, contd

- **Derivation: Resistors in series, contd**

Resistors in series and parallel, contd

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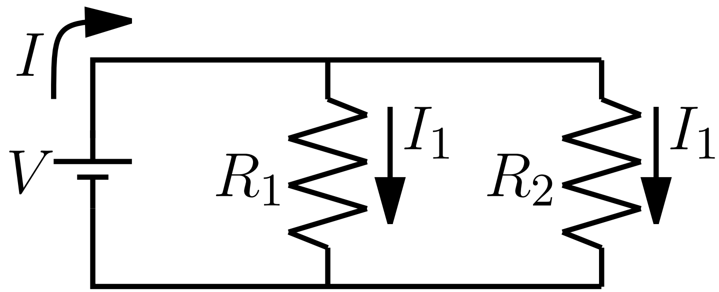
  - From Ohm's law $\boxed{V = IR}$ resistors in series equivalent to one resistor with resistance

$$\boxed{R_{eq} = R_1 + R_2 + \cdots.}$$

Resistors in series and parallel, contd

- **Derivation: Resistors in parallel**



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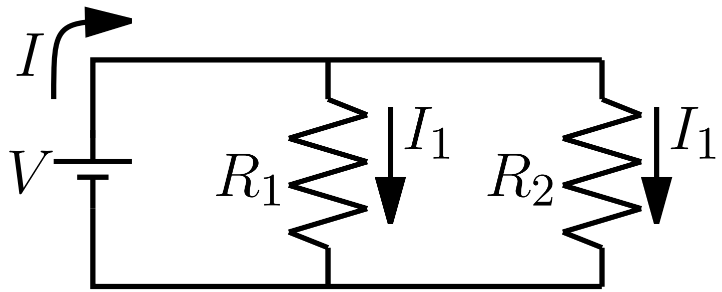

- Voltage drop must be same across both paths so, $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$.

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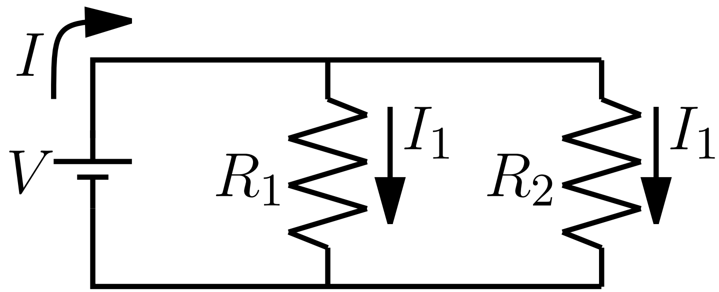

  - Voltage drop must be same across both paths so, $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$.
  - Current splits across branch,

$$\begin{aligned} I &= I_1 + I_2 \\ &= V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V}{R_{eq}}. \end{aligned}$$

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  - Resistors in parallel equivalent to one resistor with resistance R_{eq}, where

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  - Can simplify (most) complex circuits by replacing series and parallel combinations by equivalents.

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- **Interactive Quiz: PRS 10a**

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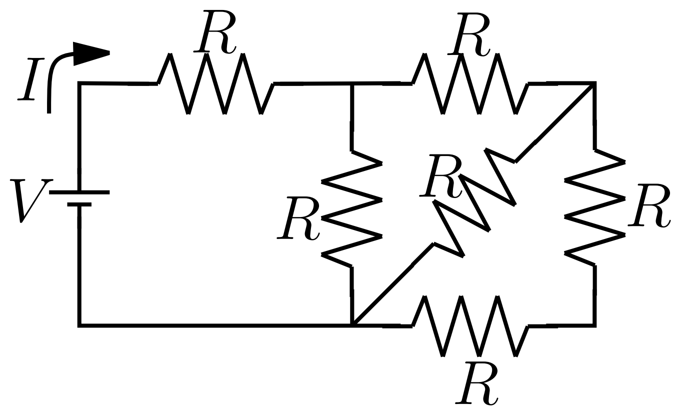
  - What is the net resistance of the circuit connected to the battery as shown?



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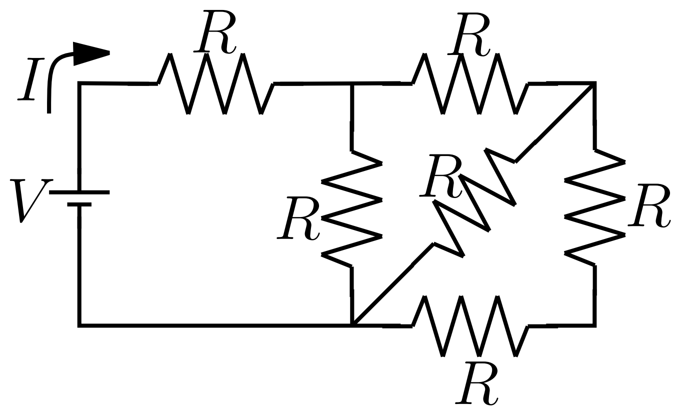

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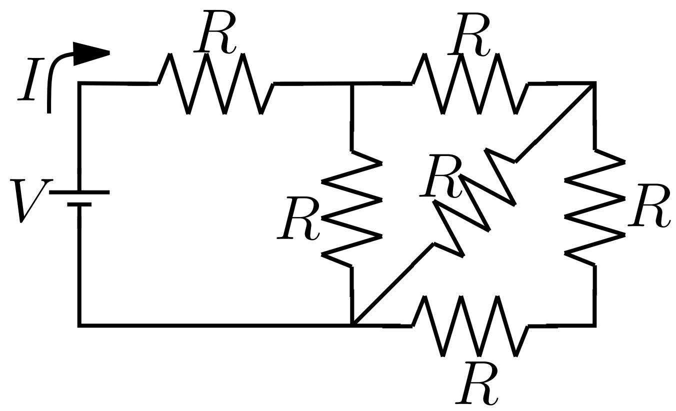

- **Solution: Pr. 11**
  - To simplify the circuit we need to find a set of resistors in series or parallel.

Resistors in series and parallel, contd

- **Solution: Pr. 11, contd**

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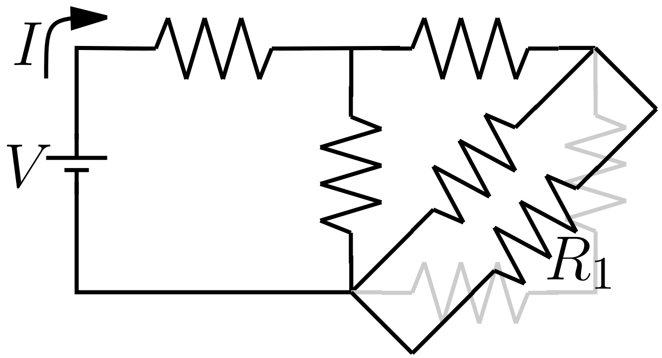
  - 2 in series at the bottom right, the equivalent resistance is $R_1 = 2R$.



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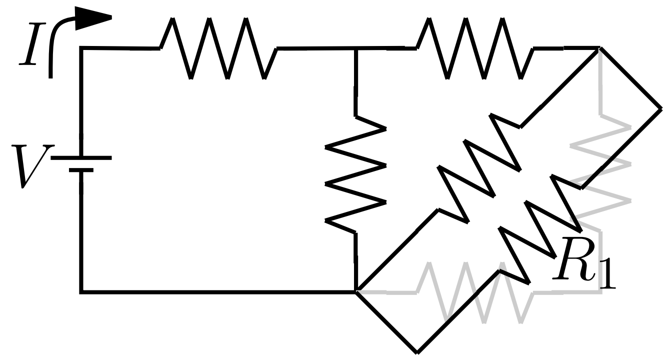
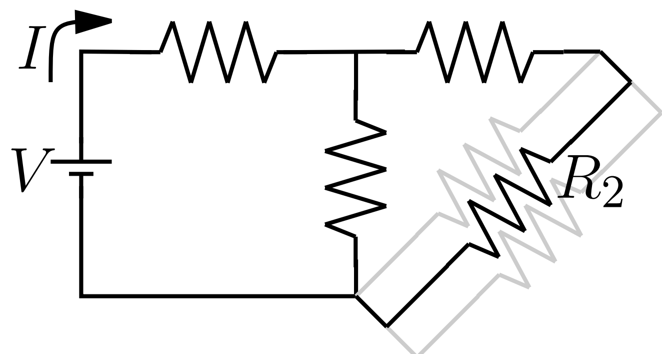


  - Parallel: $\frac{1}{R_2} = \frac{1}{R} + \frac{1}{2R}$, or $R_2 = \frac{2}{3}R$.



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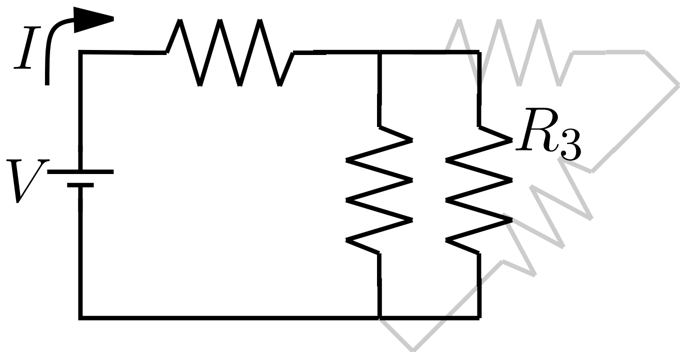

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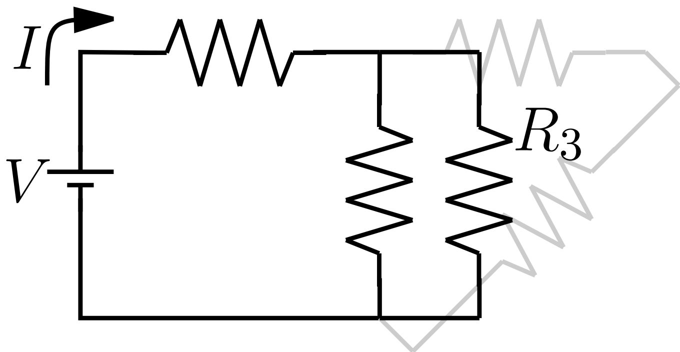
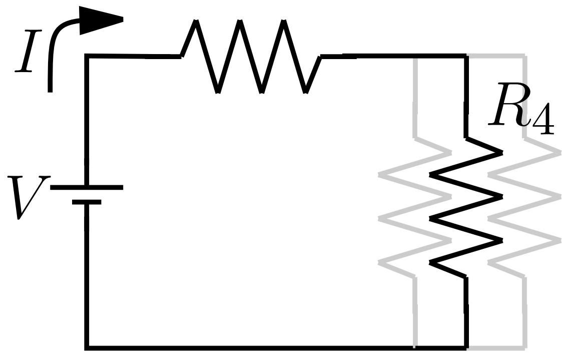


  - Parallel: $\frac{1}{R_4} = \frac{1}{R} + \frac{3}{5R}$, or $R_4 = \frac{5}{8}R$.



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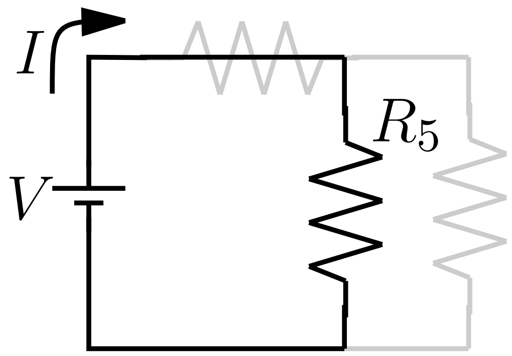

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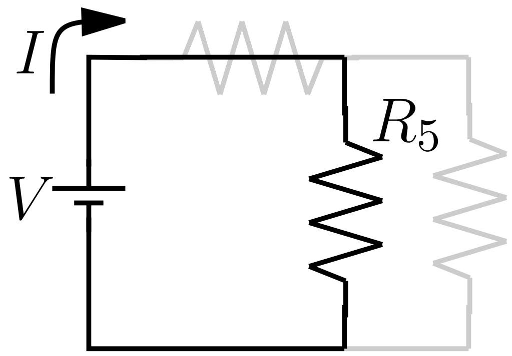

  - So the net resistance is $R_5 = \frac{13}{8}R$. $\square$

Kirchhoff’s rules [Text: Sect. 26-3]

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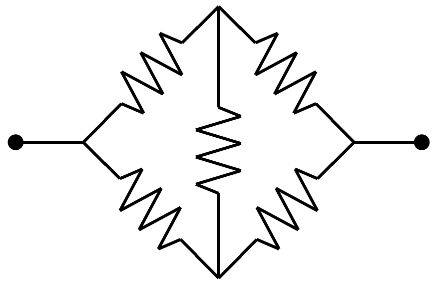

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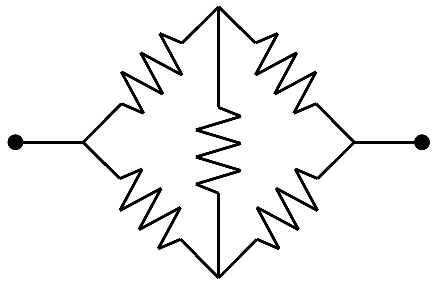

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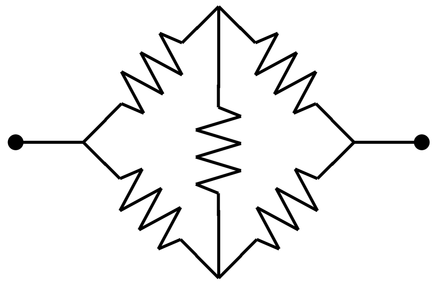

  - Can't isolate any parallel or series combinations.
  - Have to go back to basics.

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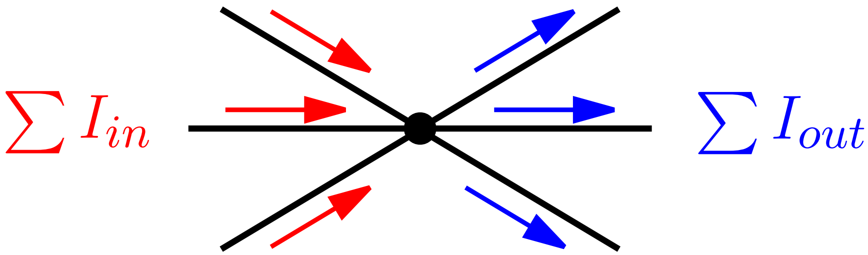

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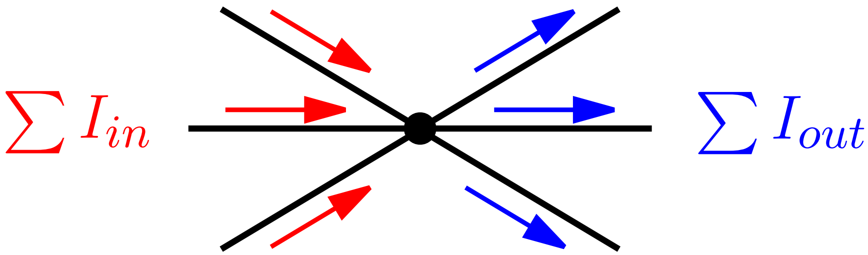

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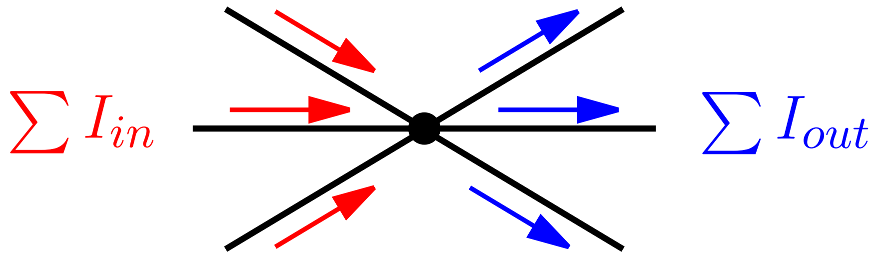

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  - Hint: sometimes you don't know the direction of the current. Then just guess and work out the answer. If $I < 0$ then you know it's going the other direction.

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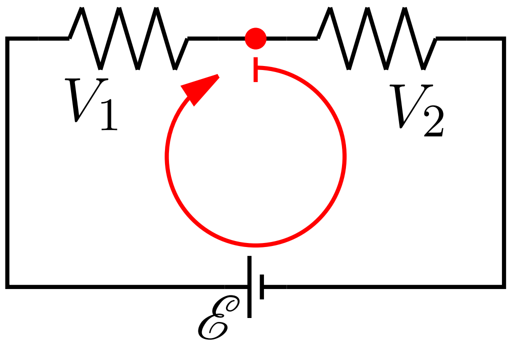

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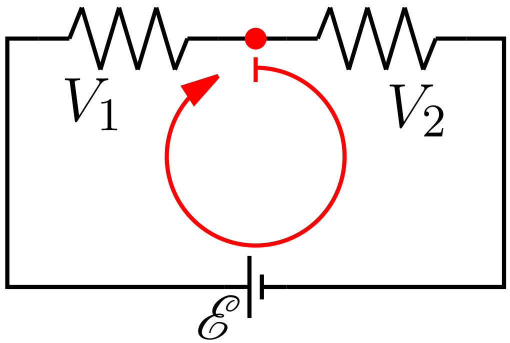

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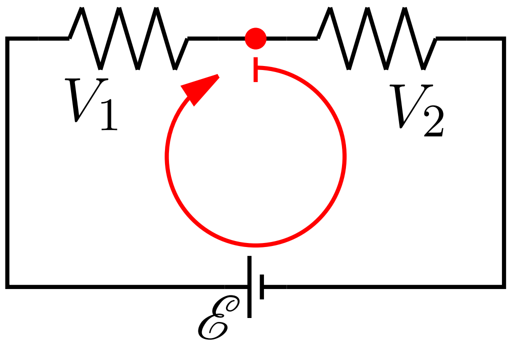

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- **Interactive Quiz: PRS 10b**

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RC Circuits [Text: Sect. 26-4]

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- **Discussion: RC Circuits**
  - Circuits can contain both resistors and capacitors.
  - Resistors limit current so it takes time for charge to build up on capacitor.
  - In steady state no current through capacitor.
  - Current decays exponentially over time,

$$I(t) = I_0 e^{-t/\tau}.$$

RC Circuits, contd

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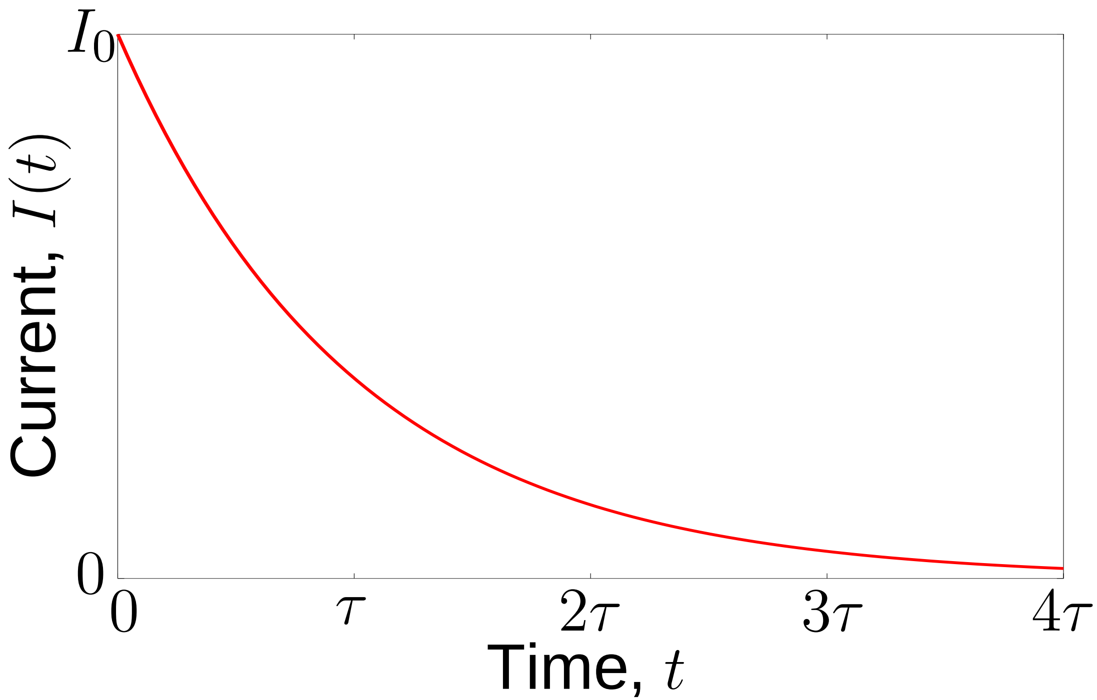

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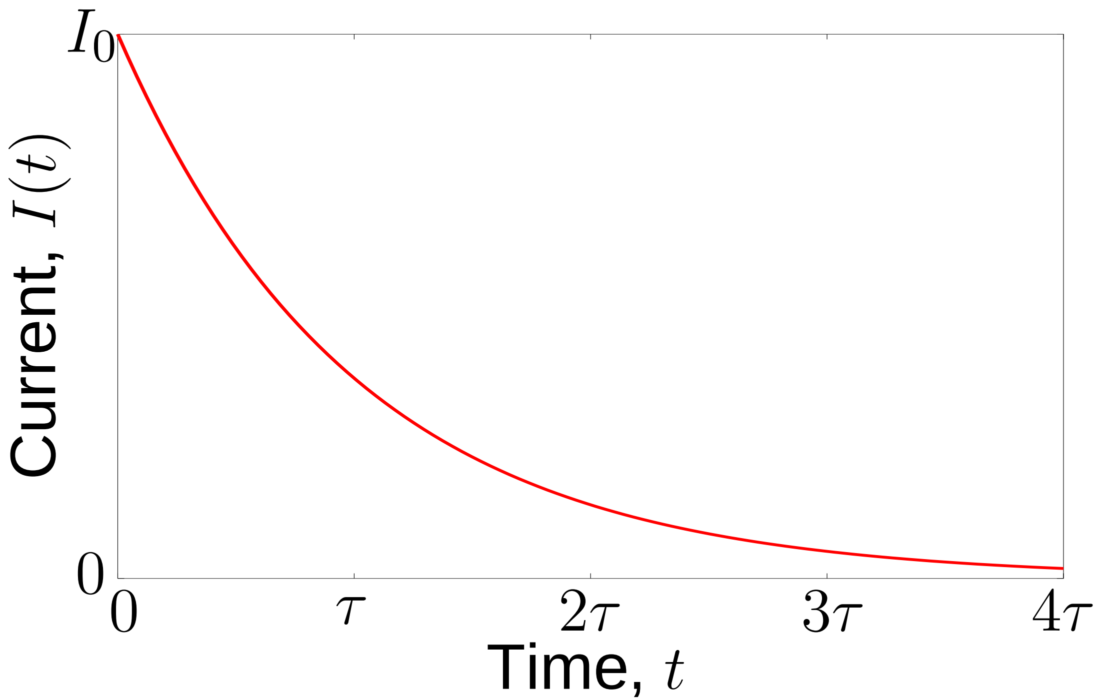

  - τ is time it takes for current to drop close (about $\frac{2}{3}$ of the way) to zero.

RC Circuits, contd

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  - Current is rate of change of charge on capacitor so charge (and voltage, $V = \frac{Q}{C}$) approach final value exponentially,

$$\boxed{\begin{aligned} Q(t) - Q_\infty &= (Q_0 - Q_\infty)e^{-t/\tau}, \\ V(t) - V_\infty &= (V_0 - V_\infty)e^{-t/\tau}. \end{aligned}}$$

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  - Use what you know about initial conditions and steady states to find I_0, Q_0, V_0, Q_∞, and V_∞.

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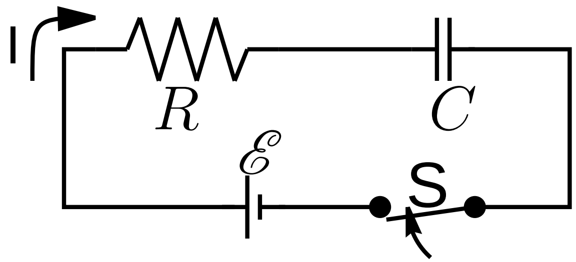
- **Example: Charging a capacitor**
  - In a circuit with a battery $\mathscr{E}$, resistor R, and capacitor C (initially uncharged) in series, what is the current I as a function of time?

RC Circuits, contd

- **Solution: Charging a capacitor**



RC Circuits, contd

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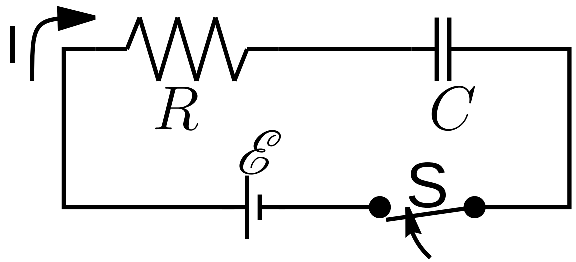

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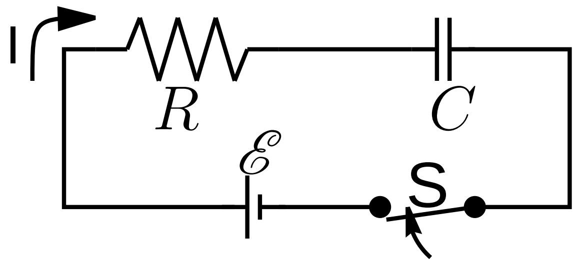

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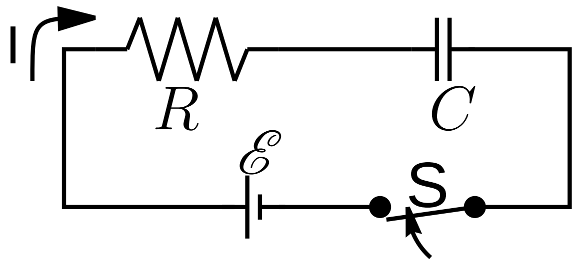

- In this simple circuit the time constant is $\boxed{\tau = RC}$.
- We know $\boxed{I(t) = I_0 e^{-t/\tau}}$ so we need to find I_0.
- I_0 is the current at the instant the capacitor starts charging (when the switch S is closed).

RC Circuits, contd

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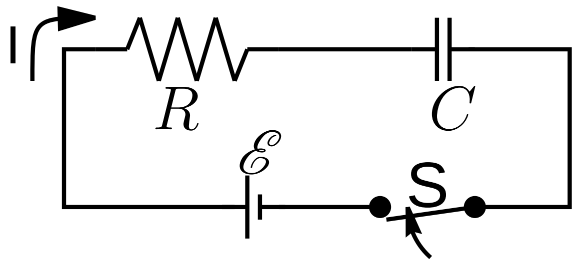

- In this simple circuit the time constant is $\boxed{\tau = RC}$.
- We know $\boxed{I(t) = I_0 e^{-t/\tau}}$ so we need to find I_0.
- I_0 is the current at the instant the capacitor starts charging (when the switch S is closed).
- Initially the capacitor is uncharged, $Q_0 = 0$. So $V_0 = \frac{Q_0}{C} = 0$.

RC Circuits, contd

- **Solution: Charging a capacitor, contd**

RC Circuits, contd

- **Solution: Charging a capacitor, contd**
  - Then the only voltage drop in the circuit is the resistor, $V = I_0 R$, and from Kirchhoff's loop rule,

$$\mathscr{E} = I_0 R.$$

RC Circuits, contd

- **Solution: Charging a capacitor, contd**
  - Then the only voltage drop in the circuit is the resistor, $V = I_0 R$, and from Kirchhoff's loop rule,

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  - So $I_0 = \frac{\mathcal{E}}{R}$ and the current as a function of time is

$$I(t) = \frac{\mathcal{E}}{R} e^{-t/(RC)}. \quad \square$$

RC Circuits, contd

- **Solution: Charging a capacitor, contd**
  - Then the only voltage drop in the circuit is the resistor, $V = I_0 R$, and from Kirchhoff's loop rule,

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- **Interactive Quiz: PRS 10d**

End

- **Practice Problems:**
  - Ch. 26: Q. 3, 7, 9, 11, 13, 15, 17, 19.
  - Ch. 26: Pr. 1, 3, 5, 7, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 41, 43, 45, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83.

End

- **Practice Problems:**
  - Ch. 26: Q. 3, 7, 9, 11, 13, 15, 17, 19.
  - Ch. 26: Pr. 1, 3, 5, 7, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 41, 43, 45, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83.
- **Interactive Quiz: Feedback**

End

- **Practice Problems:**
  - Ch. 26: Q. 3, 7, 9, 11, 13, 15, 17, 19.
  - Ch. 26: Pr. 1, 3, 5, 7, 13, 15, 17, 19, 21, 23, 25, 27, 29, 31, 33, 35, 37, 41, 43, 45, 63, 65, 67, 69, 71, 73, 75, 77, 79, 81, 83.
- **Interactive Quiz: Feedback**
- **Tutorial Question: tut10**