# UBC Physics 102

## *Lecture 6*

Rik Blok

# Outline

- ▷ Electric flux
- ▷ Gauss's law
- ▷ Applications
- ▷ End

# Electric flux [Text: Sect. 22-1]

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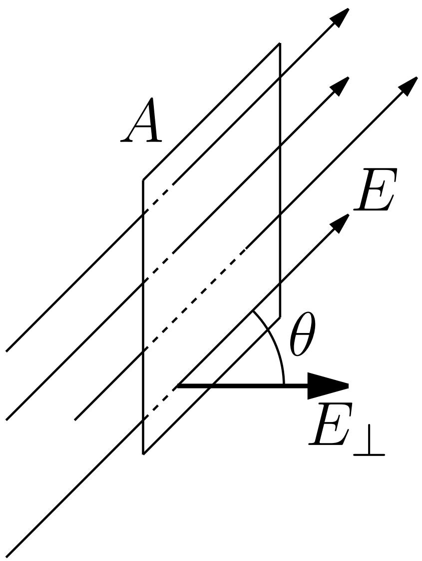

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- **Interactive Quiz: PRS 06a**

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  - So nonzero flux means volume encloses charge.

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  - $Q_{\text{encl}}$ doesn't depend on positions or configuration of charges or on any charges outside surface (but electric field does).

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  - Perpendicular sections have constant $E_\perp$ so $\Phi_E = E_\perp A$ is easy to compute.

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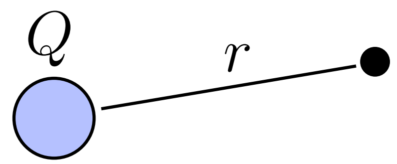
  - Can derive Coulomb's law from Gauss's law. Want to determine electric field at a distance $r$ from a point charge $Q$.



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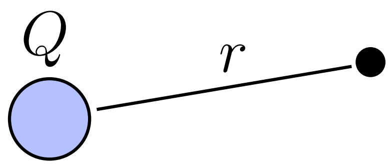


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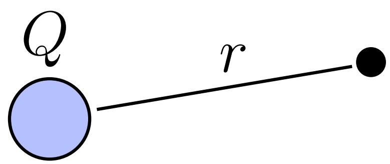


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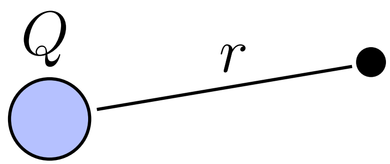


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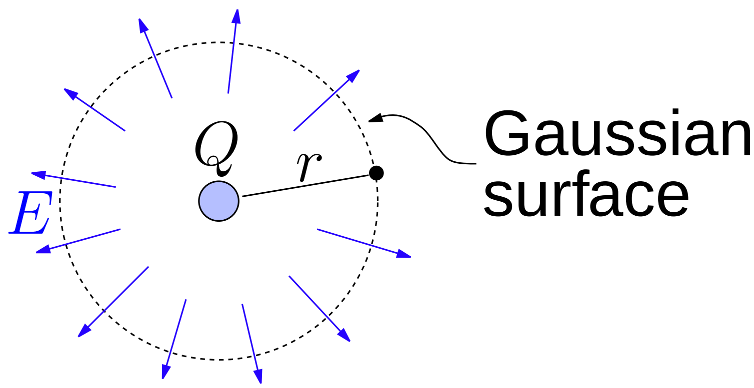

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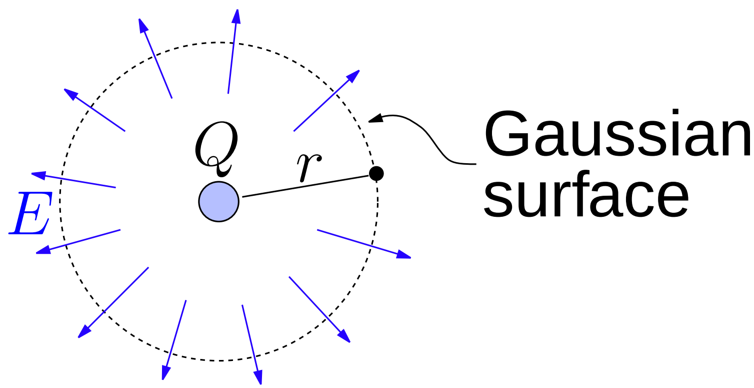


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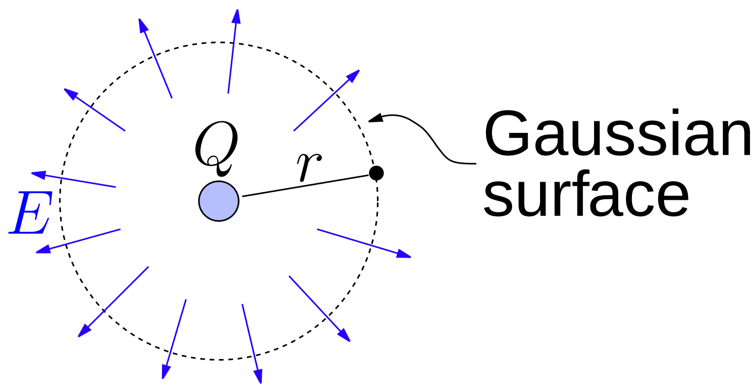


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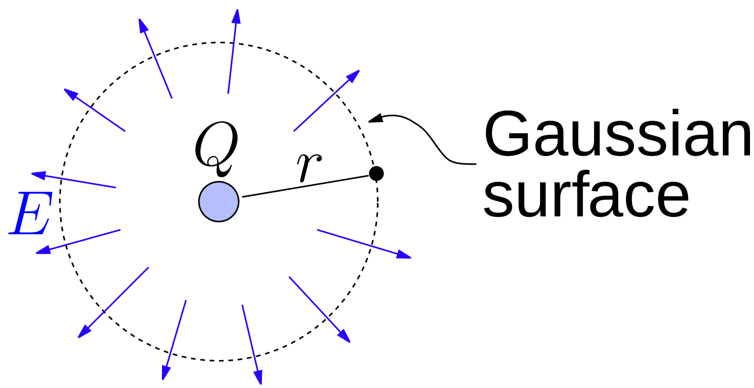


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- Gauss's law: $Q/\epsilon_0 = E(4\pi r^2)$ or

$$E = \frac{Q}{4\pi\epsilon_0 r^2} = \frac{kQ}{r^2}. \quad \square$$

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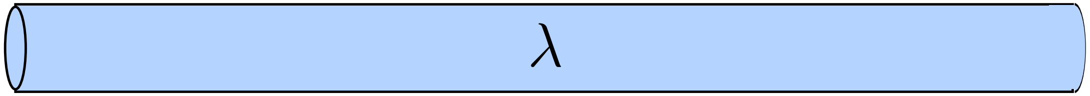
- **Example: Long, uniformly charged line**
  - A very long, straight wire possesses a uniform charge per unit length $\lambda$. Calculate the electric field at a distance $r$ from the wire.



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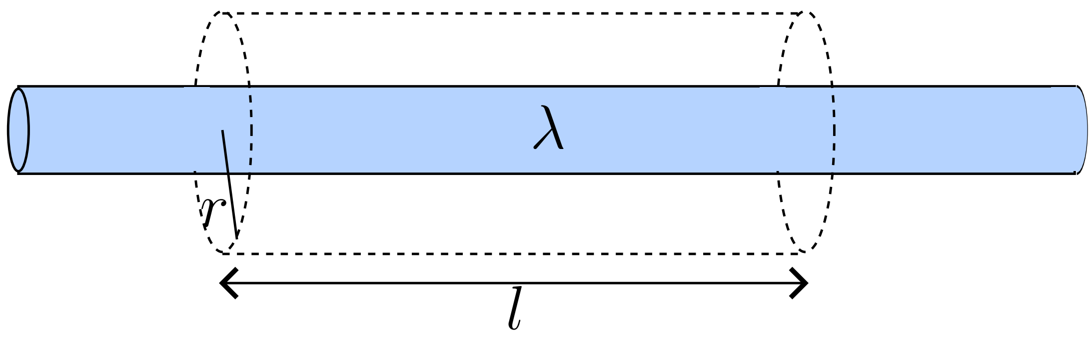

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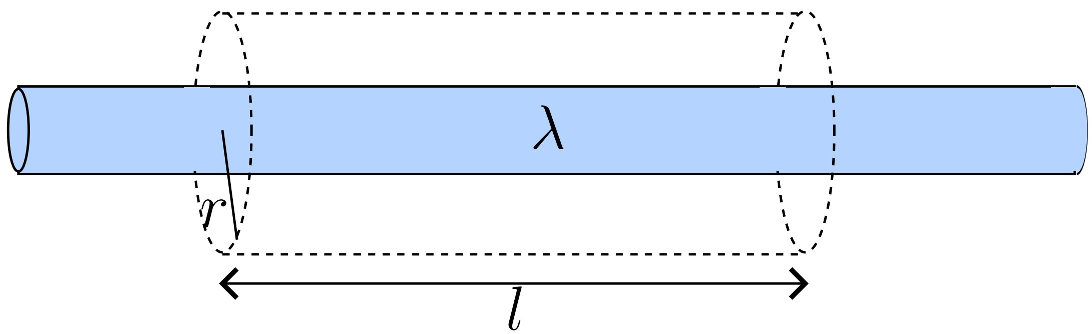


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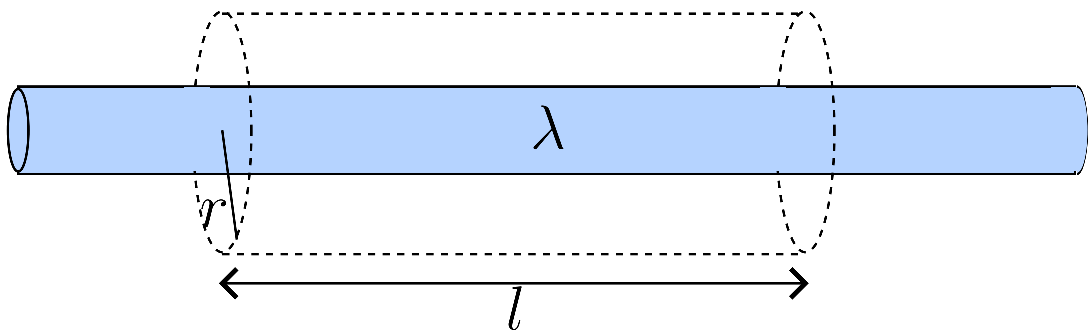


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  - Enclosed charge: $Q_{\text{encl}} = \lambda l$.

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  - Flux: $(E_\perp A)_{\rm side} = E\,2\pi r l$, $(E_\perp A)_{\rm ends} = 0$.
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  - Could have solved with Coulomb's law but would have had to integrate.

# End

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  - Ch. 22: Pr. 3, 5, 7, 9, 17, 21, 23, 25, 27, 29, 31, 33, 39, 41, 43, 45, 47, 49.

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- **Tutorial Question: tut06**