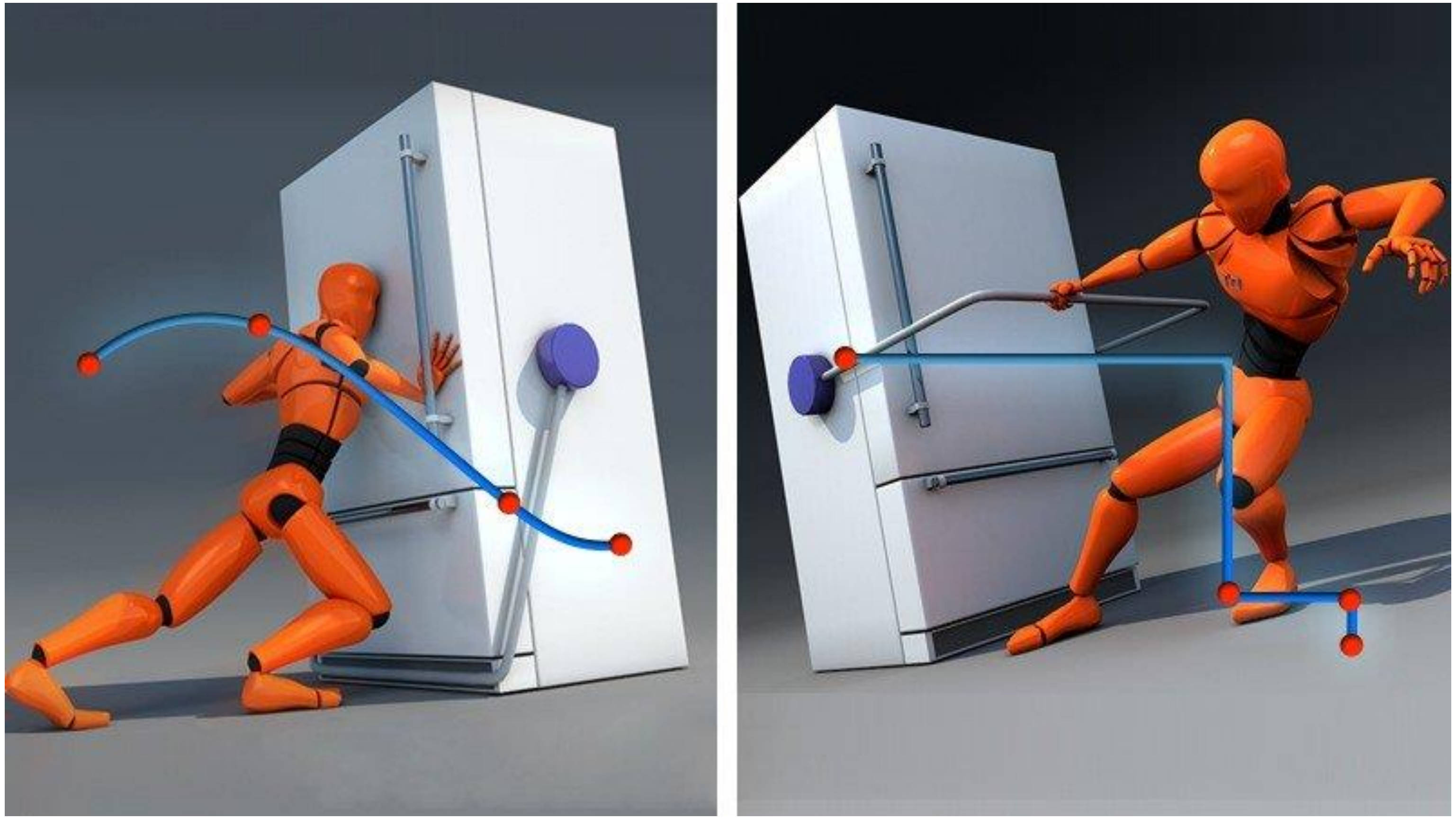
Helge Rhodin

CPSC 427
Video Game Programming

Curves and Animation



https://www.pluralsight.com/blog/film-games/stepped-vs-spline-curves-blocking-animation

Overview

1. ***Animation basics***

2. ***Curves***

Logistics

- ***Team presentations on Tuesday (9th)***

- ***Guest lecture on Thursday (11th)***
  - *Craig Peters (EA)*
  - *Debugging and peer review*

- Upcoming lectures
  - *Testing and User Studies*
  - *Composite transformations and inverse kinematics animation*

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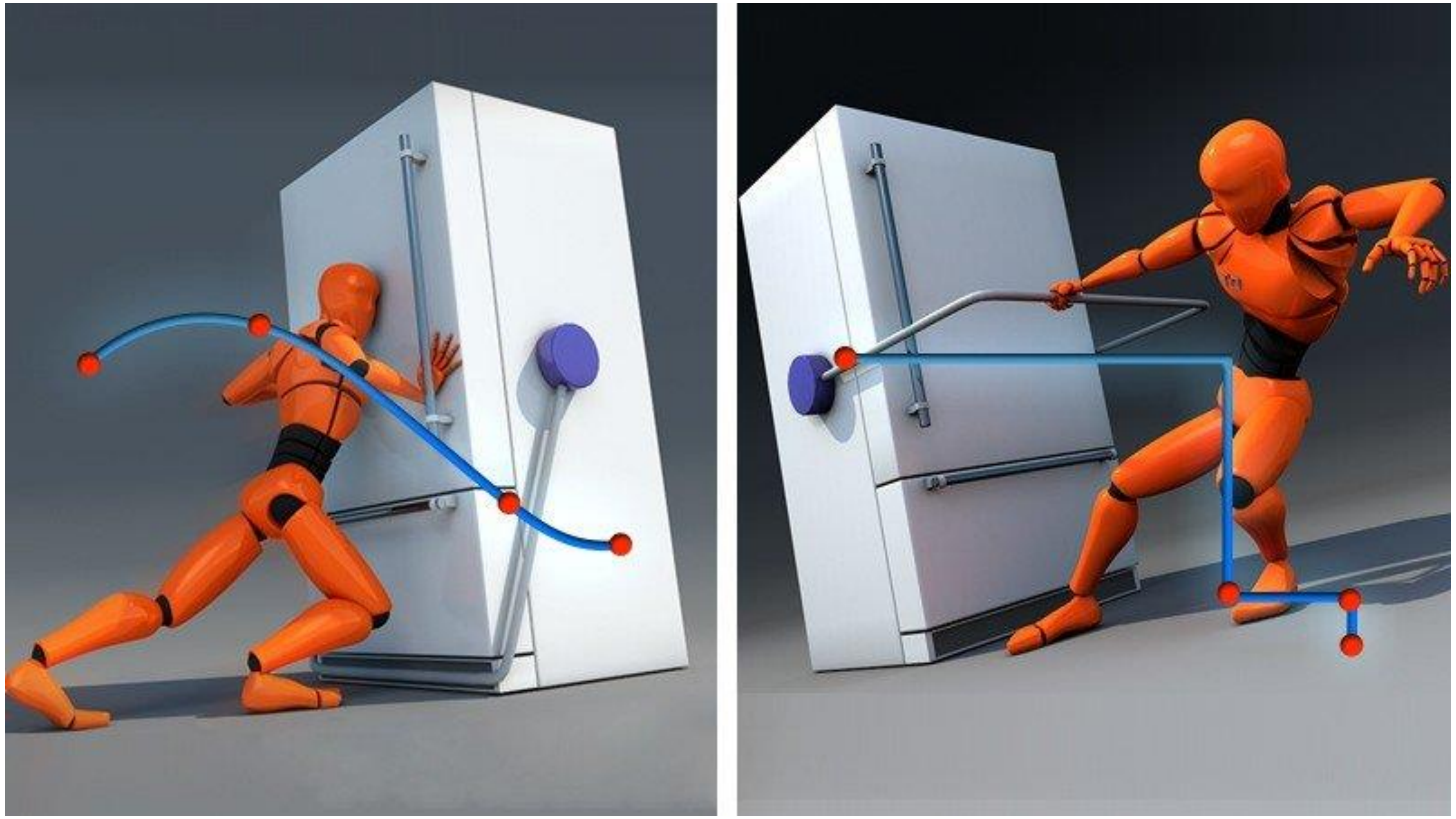
Video Game Programming

Curves and Animation



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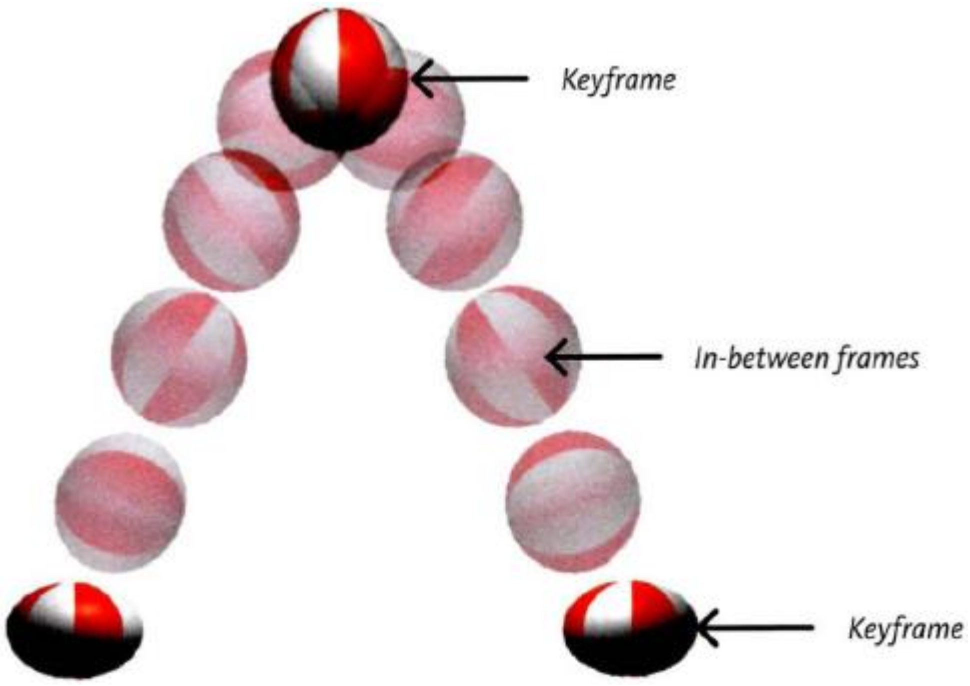
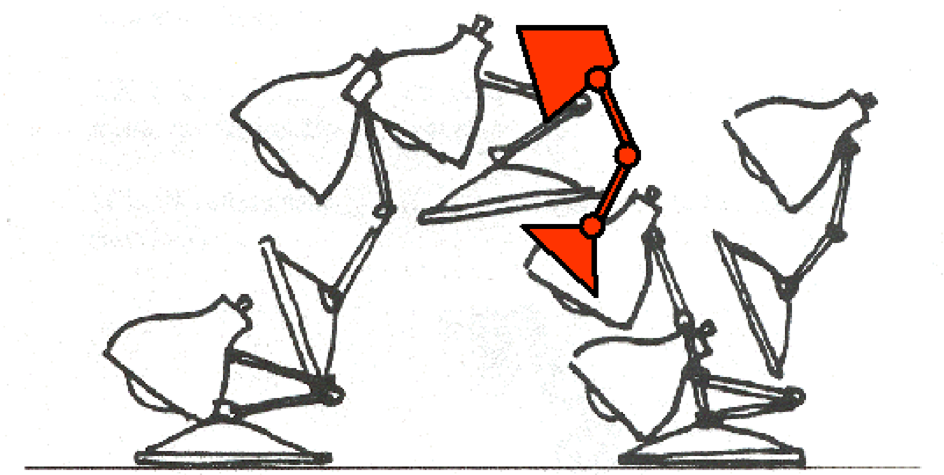
Keyframe animation





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Recap: Line equation

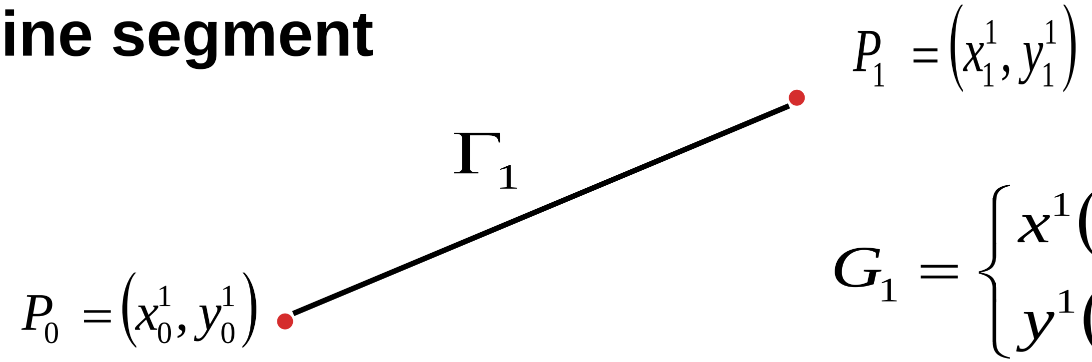
Parametric form

- 3D: x, y, and z are functions of a parameter value t

$$C(t) := \begin{pmatrix} P_y^0 \\ P_x^0 \end{pmatrix} t + \begin{pmatrix} P_y^1 \\ P_x^1 \end{pmatrix} (1-t)$$

What things can we interpolate?

Line segment



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} t \in [0,1]$$

Interpolating general properties

- *position*
- *aspect ratio?*
- *scale*
- *color*
- *What else?*

$$C(t) := \begin{pmatrix} P_y^0 \\ P_x^0 \end{pmatrix} t + \begin{pmatrix} P_y^1 \\ P_x^1 \end{pmatrix} (1-t)$$

$$s^0 \qquad s^1$$

$$c^0 \qquad c^1$$

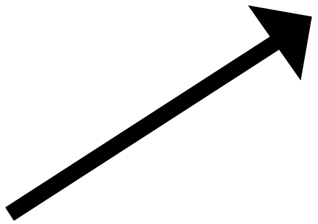
Barycentric coordinates / interpolation

Other Parametric Functions

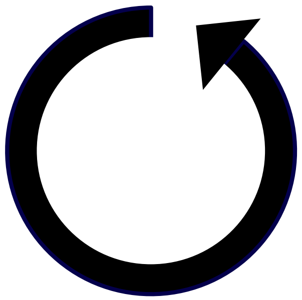
$$C(t) := \begin{pmatrix} P_y^0 \\ P_x^0 \end{pmatrix} t + \begin{pmatrix} P_y^1 \\ P_x^1 \end{pmatrix} (1-t)$$

$$C(t) := \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

Line segment



Circle (arc)



?

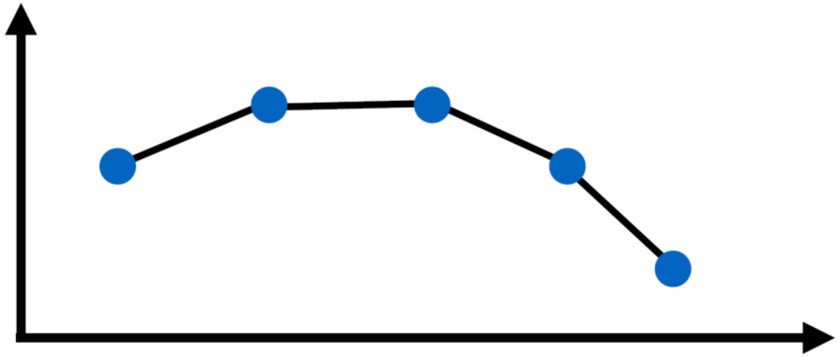
Splines

Splines

Segments of simple functions

$$f(x) = \begin{cases} f_1(x), & \text{if } x_1 < x \leq x_2 \\ f_2(x), & \text{if } x_2 < x \leq x_3 \\ \vdots & \vdots \\ f_n(x), & \text{if } x_n < x \leq x_{n+1} \end{cases}$$

E.g., linear functions



Splines – Free Form Curves

Usually parametric

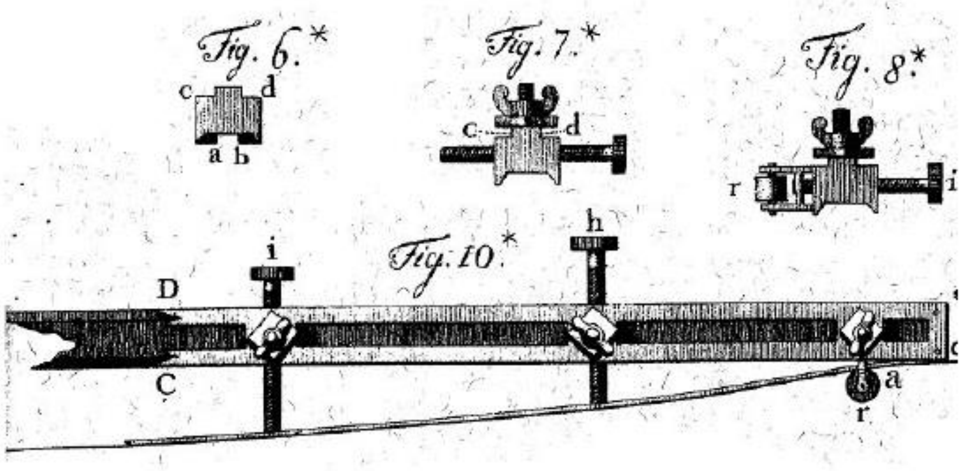
- $C(t)=[x(t),y(t)]$ or $C(t)=[x(t),y(t),z(t)]$

Description = basis functions + coefficients

$$C(t)=\sum_{i=0}^{n} P_i B_i(t)=(x(t),y(t))$$

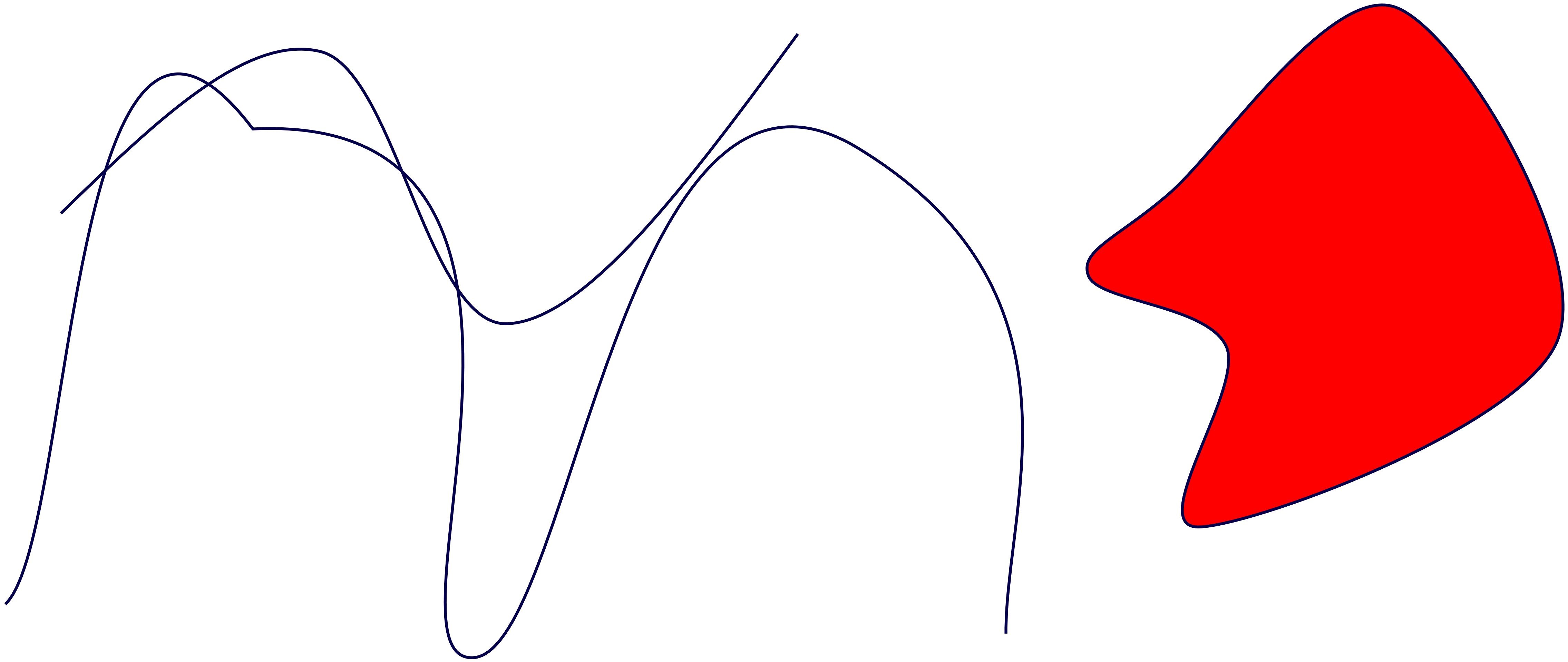
$$x(t)=\sum_{i=0}^{n} P_i^x B_i(t)$$

$$y(t)=\sum_{i=0}^{n} P_i^y B_i(t)$$

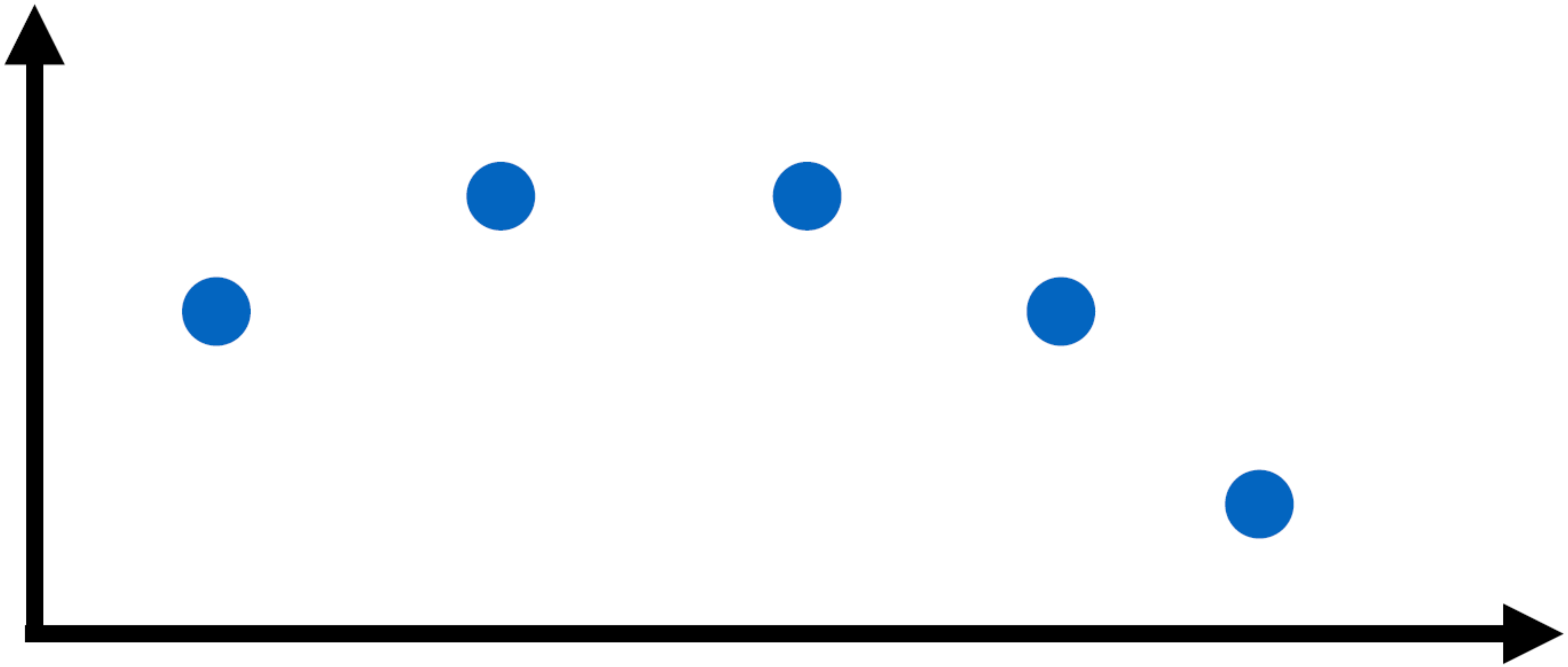


- Same basis functions for all coordinates

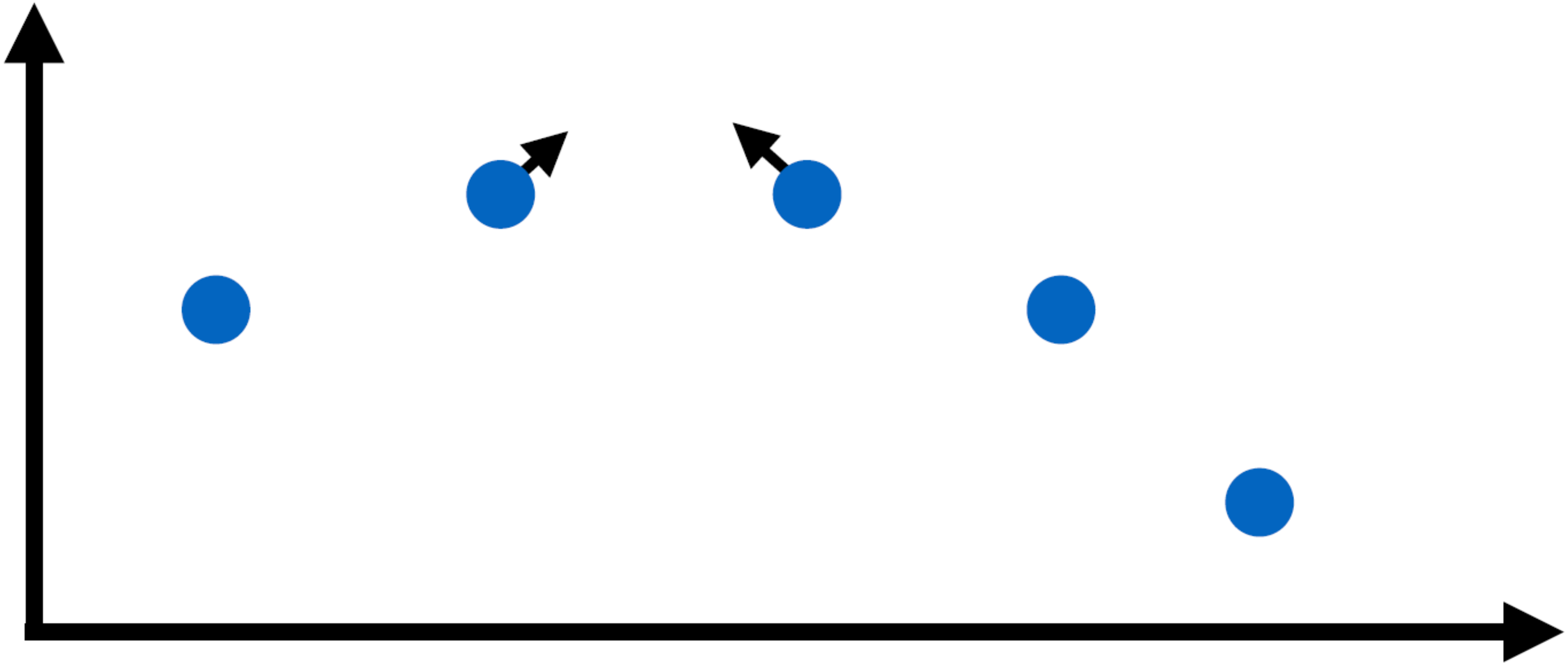
Curves



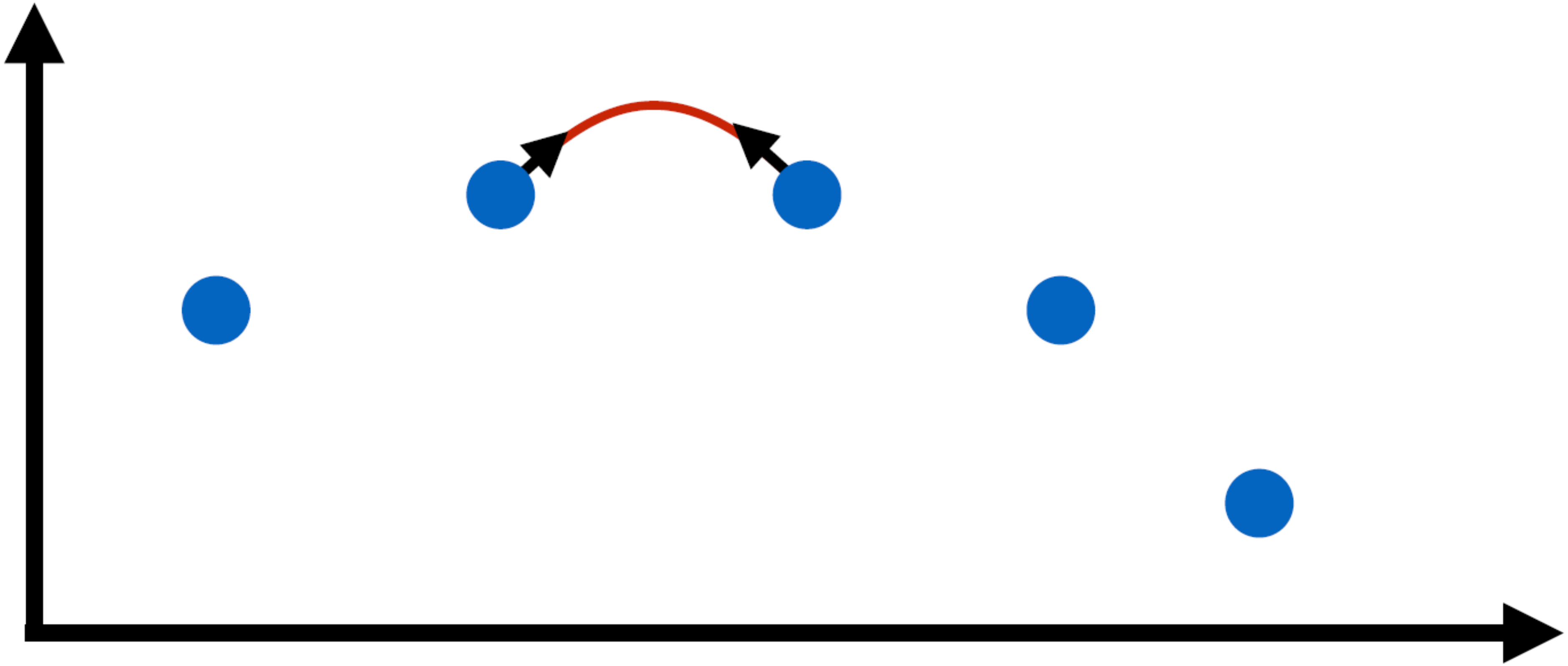
Smooth curve



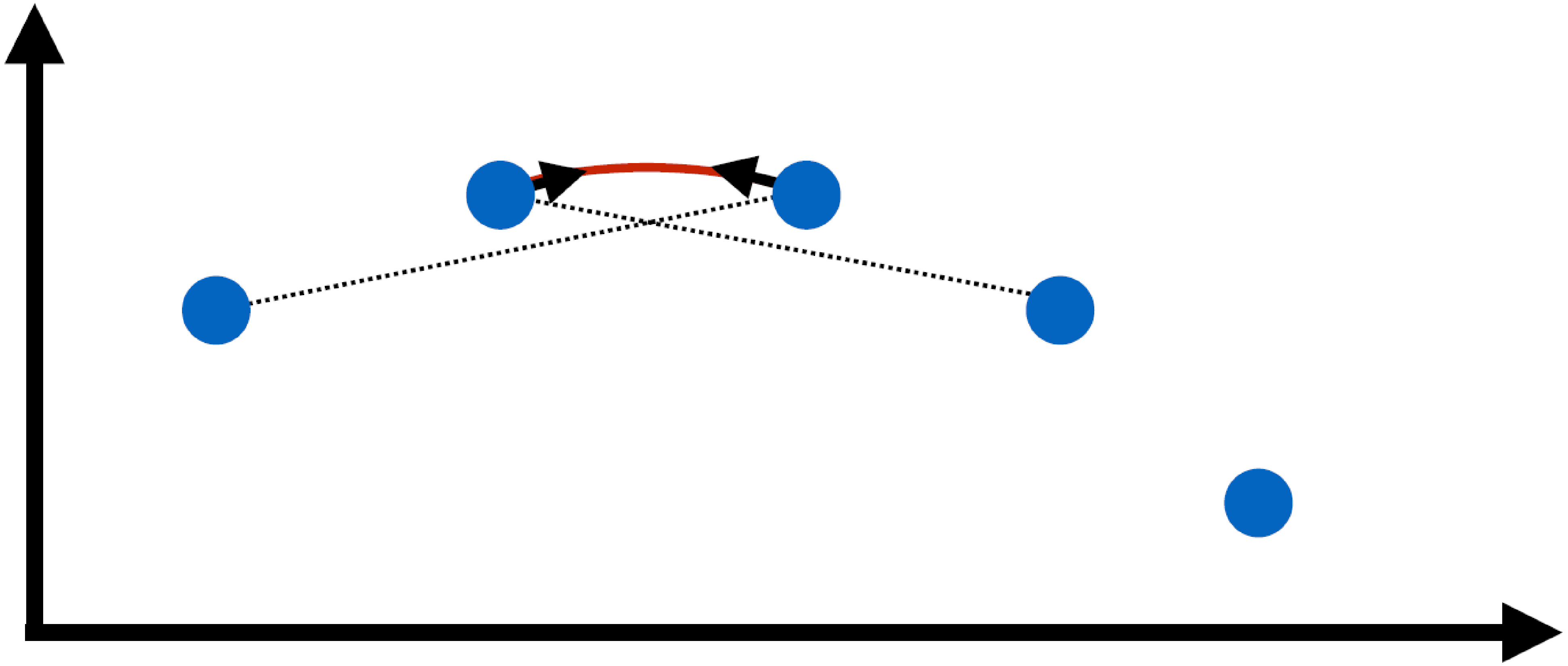
Smooth curve



Smooth curve



Smooth curve



Hermite Cubic Basis

Geometrically-oriented coefficients

- 2 positions + 2 tangents

Require $C(0)=P_0$, $C(1) = P_1$, $C'(0)=T_0$, $C'(1)=T_1$

Derivatives of C at 0 and 1

Define basis functions, one per requirement

$$C(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$

Hermite Basis Functions

$$C(t) = P_0 h_{00}(t) + P_1 h_{01}(t) + T_0 h_{10}(t) + T_1 h_{11}(t)$$

To enforce** $C(0)=P_0$, $C(1) = P_1$, $C'(0)=T_0$, $C'(1)=T_1$ **basis should satisfy

$$h_{ij}(t) : i, j = 0,1, t \in [0,1]$$

curve	$C(0)$	$C(1)$	$C'(0)$	$C'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

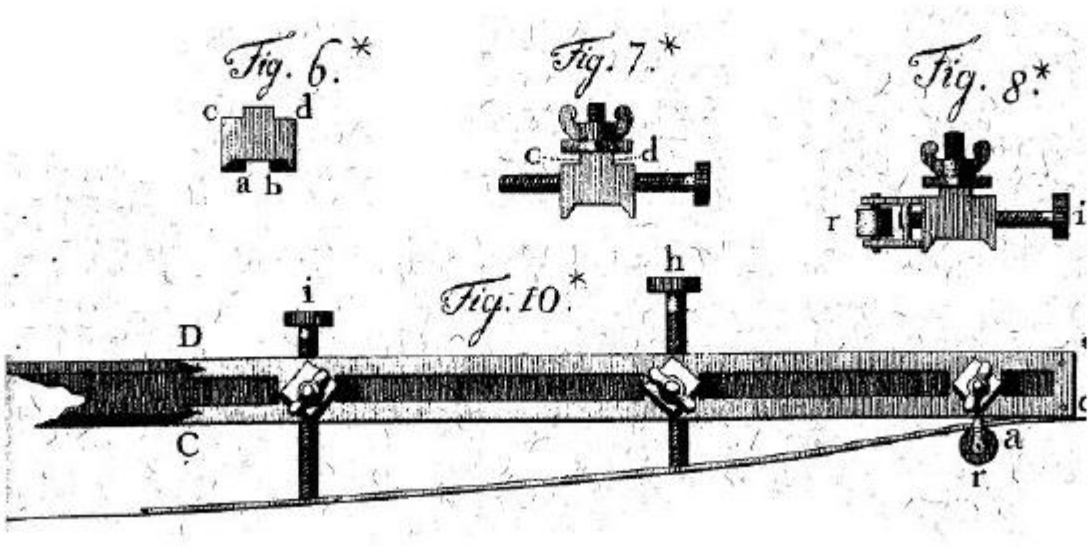
$h_{00}'(0) = h_{00}'(0) = 0$

$h_{00}(0) = 1$

Splines – Free Form Curves

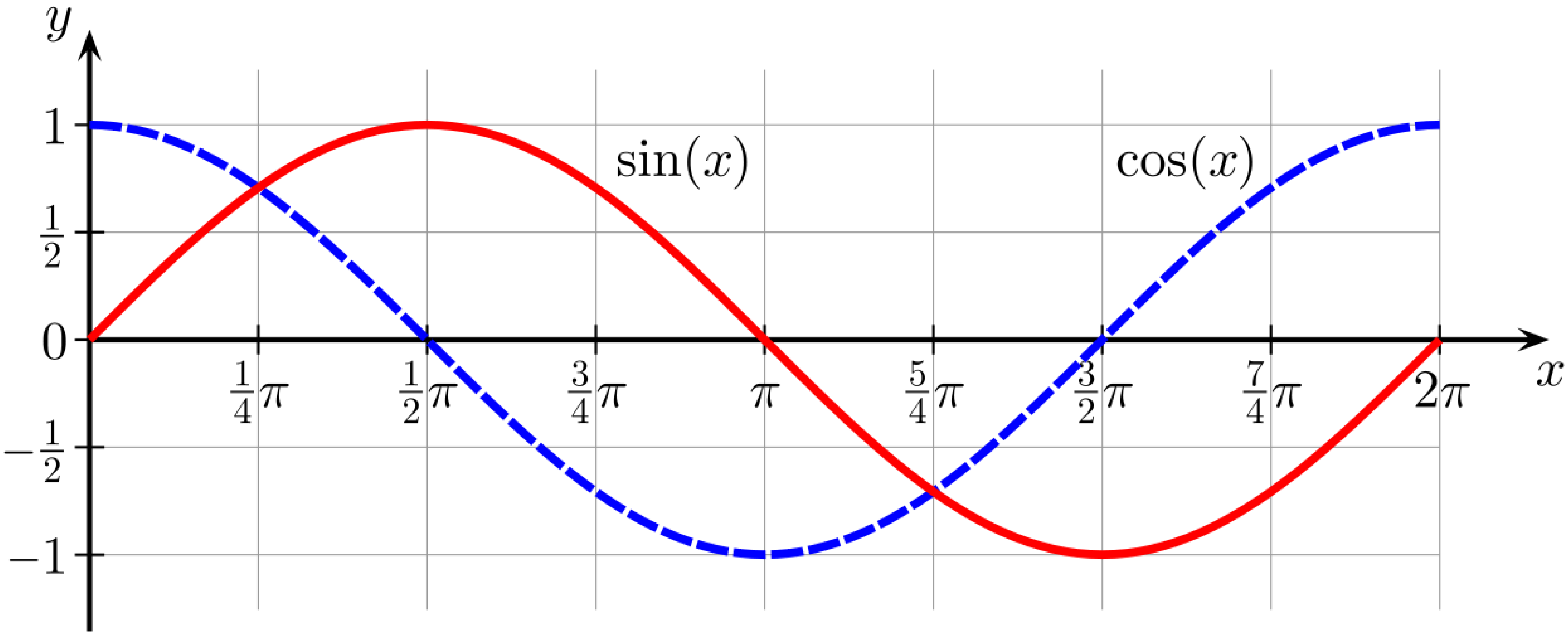
Geometric meaning of coefficients (base)

- Approximate/interpolate set of positions, derivatives, etc..



Will see one example

Possible solution?



Hermite Cubic Basis

Can satisfy with cubic polynomials as basis

$$h_{ij}(t) = a_3 t^3 + a_2 t^2 + a_1 t + a_0$$

Obtain - solve 4 linear equations in 4 unknowns for each basis function

$$h_{ij}(t): i, j = 0,1, t \in [0,1]$$

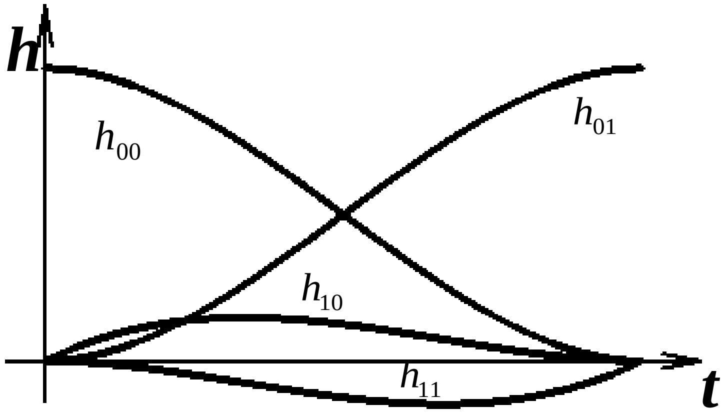
curve	$C(0)$	$C(1)$	$C'(0)$	$C'(1)$
$h_{00}(t)$	1	0	0	0
$h_{01}(t)$	0	1	0	0
$h_{10}(t)$	0	0	1	0
$h_{11}(t)$	0	0	0	1

Hermite Cubic Basis

Four cubic polynomials that satisfy the conditions

$$h_{00}(t) = t^2(2t-3)+1 \qquad h_{01}(t) = -t^2(2t-3)$$

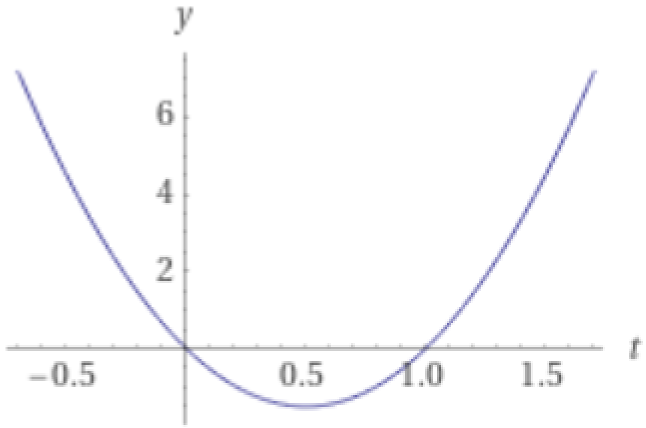
$$h_{10}(t) = t(t-1)^2 \qquad h_{11}(t) = t^2(t-1)$$



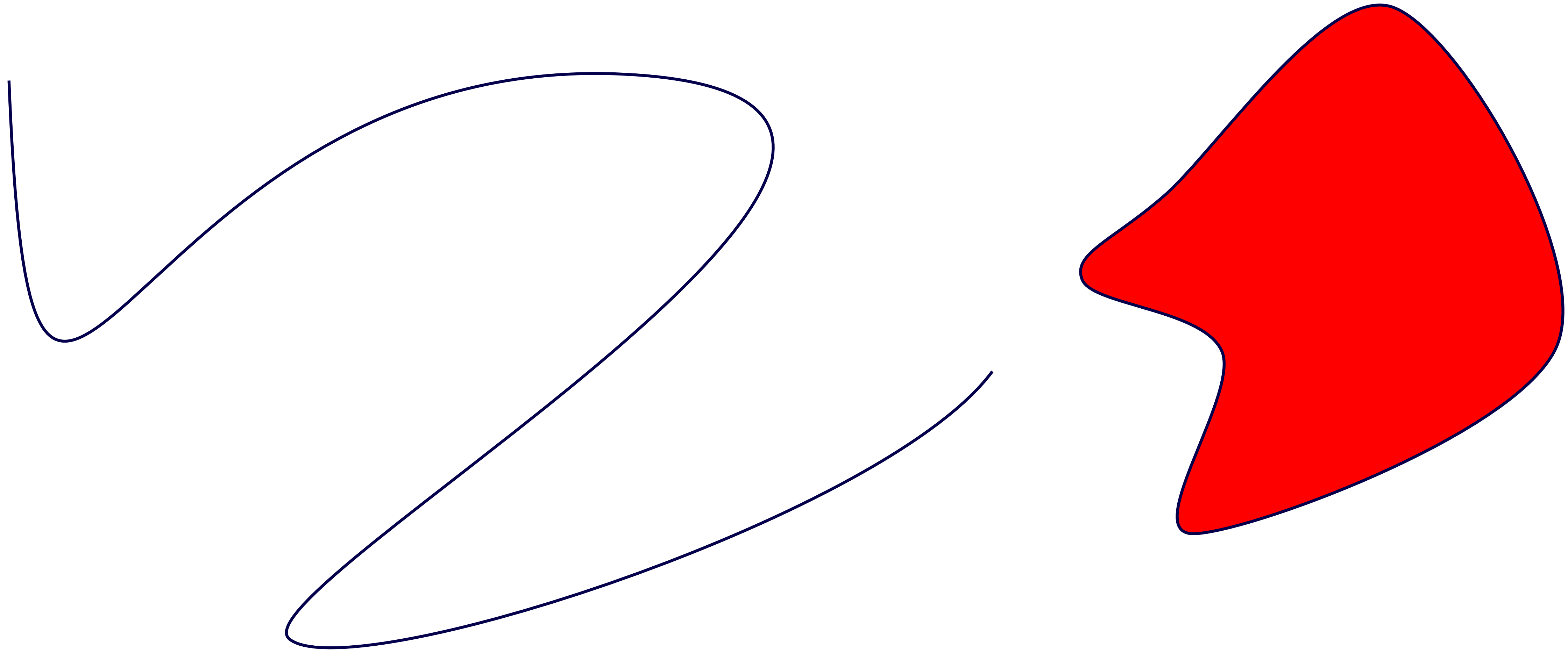
Derivative of h00

$6(-1+t)t$

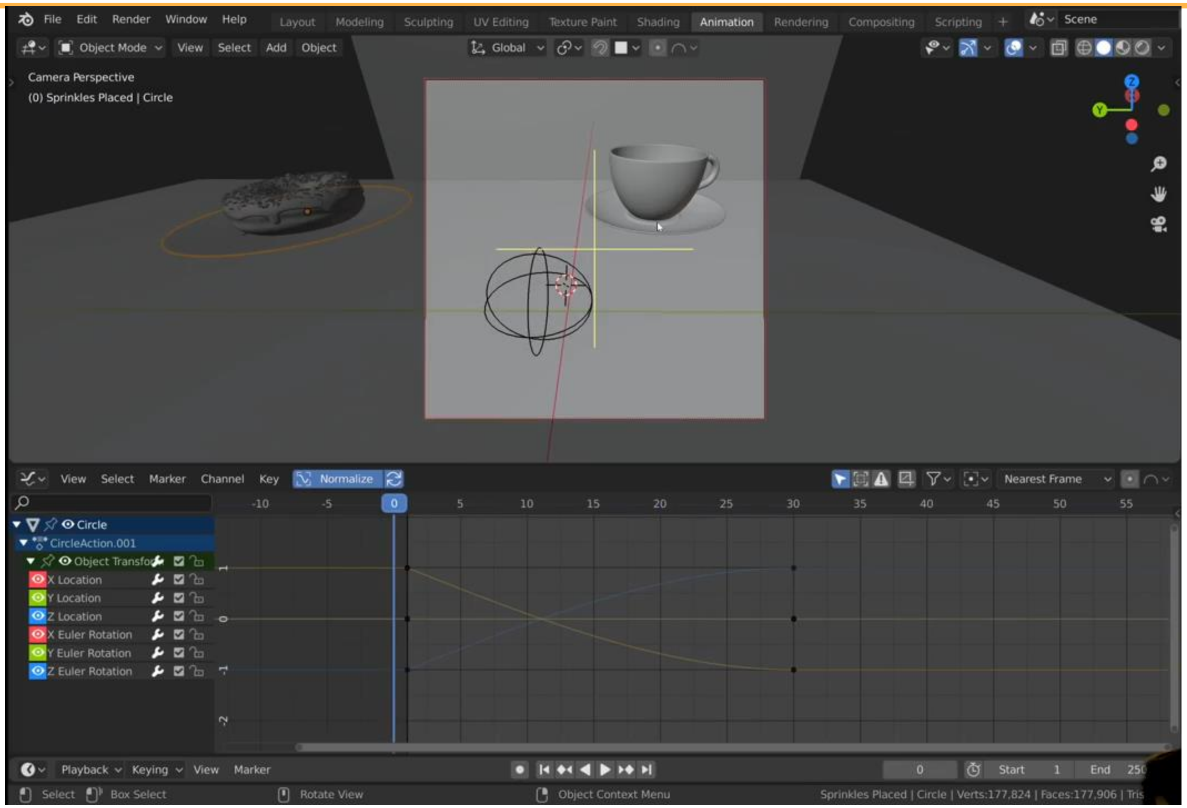
Plots:



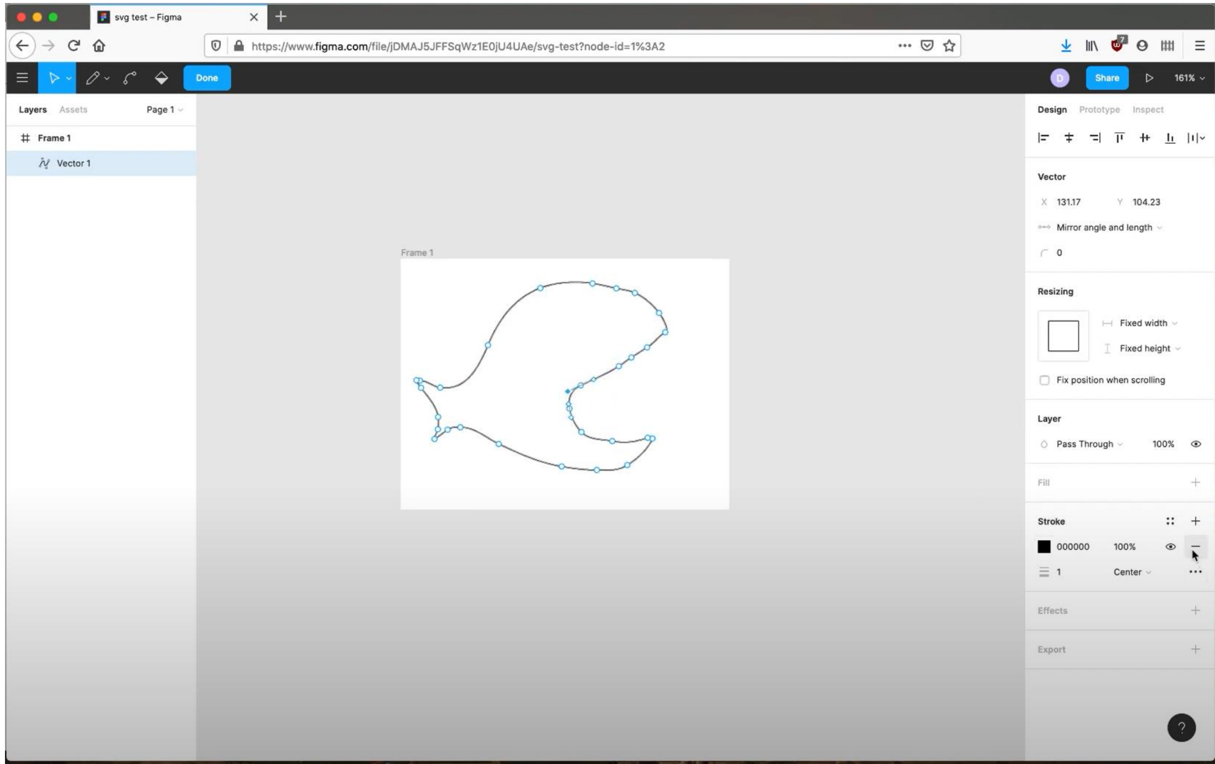
Curves



Applications:
Keyframe animation & mesh creation



https://www.youtube.com/watch?v=LLImJxTyNw



Dave’s Tutorial