

# Better Cloth Through Bulletproof Time Integration and Physics-Aware Subdivision

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## Abstract

*We present a novel time integration technique which employs an element-by-element method to avoid the use of a nonlinear solver, and a quasi-static method for extracting intricate details out of coarse cloth meshes.*

Categories and Subject Descriptors (according to ACM CCS): I.3.7 [Computer Graphics]: Three-Dimensional Graphics and Realism, I.3.5 [Computer Graphics]: Physically based modeling

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## 1. Introduction

The ongoing challenge of cloth simulation in both interactive applications and film (where animators generally use iterative design, rerunning a simulation many times with different parameters) is speed. Current methods to allow large time steps do not have guarantees on stability and/or consistency with respect to physical parameters; we correct this with a new "bulletproof" time integration scheme. We also propose a physics-aware subdivision scheme which adds missing wrinkle and fold detail to coarse mesh simulations in a quasi-static way, making the use of faster and more easily-controlled coarse meshes more practical.

## 2. Bulletproof Time Integration

Linearized implicit methods such as that of [BW98] provide a framework for running simulations with larger time steps than explicit methods but still can be unstable above a certain unknown timestep. Certain instability issues arise in that the solution to nonlinear problems generally depends on the performance of a Newton-based iterative framework which is not guaranteed to converge. Our method takes a slightly different approach in that we attempt to break the problem down into a set of smaller subproblems, solve each of these in isolation and then find a global solution as close as possible to the combination of all the individual solutions.

Our system acts as a post-processor, taking the resulting positions and velocities of an arbitrary method's forward integration of external forces (such as gravity) as inputs. These

are fed into our 3 step system composed of a global force estimate, element-by-element responses and, finally, a global stitching step.

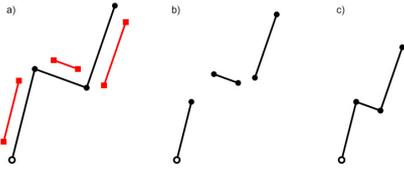
The first step is a standard step of time integration to enforce maximum strain constraints on edges in the mesh using the Lagrange multiplier method of [Bar96]. This step serves as an initial guess for the forces acting on each element (e.g. spring, or FEM triangle) for the timestep.

The first step may not produce a stable response and so the second step uses the global force estimate of the first step to compute the nonlinear response of each element in isolation, which can typically be done in analytic fashion, guaranteeing stability. Because each element is worked on individually, it is very likely that elements will disagree on the final position of a shared node. Thus, we are left with a set of separated elements which have each moved in isolation and in accordance only with the local forces acting on them directly.

The third step reconnects all of the individual elements, stitching the system back together with a linear least squares solve based on an error metric which does not violate previously enforced stretch constraints.

Figure 1 shows an example of our method broken up into the three major steps and being applied to a mass spring system consisting of a single chain of mass points connected together in succession.

This three-step approach is physically correct and order-independent (unlike standard strain limiting) because we



**Figure 1:** a) Mass spring system with 1 constrained point (white point). Red lines show the maximum allowable lengths of each spring. At this point an initial guess of forces is calculated b) The impulse forces from step 1 are applied to each spring individually. Shared nodes disagree on their final positions since they are being pulled in different directions. c) Step 3 stitches the system back together, respecting the constrained points and lengths of springs.

compute a valid global estimate of forces and correct element-by-element responses. It is guaranteed to be stable since the element computations are stable and the global stitching step confines nodes to the convex hull of the element responses and is linear, hence there are no convergence issues.

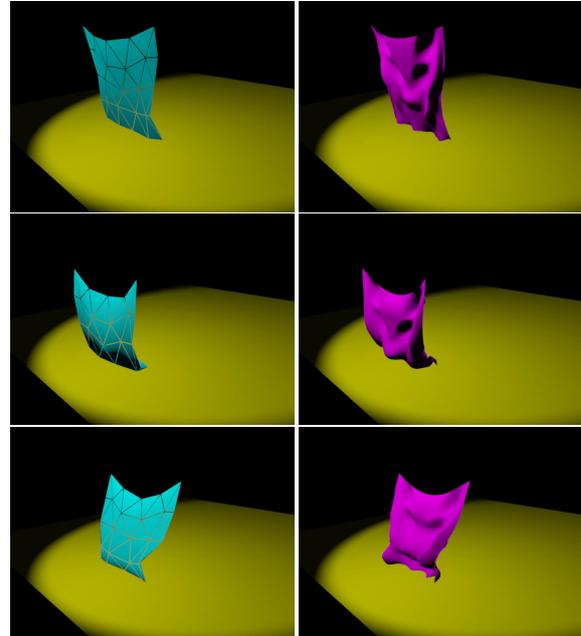
### 3. Physics Aware Subdivision

Recall that subdivision is a model for splines, which originated as the actual mechanical equilibrium of a piece of thin wood or metal subjected to point constraints, where its combination of elastic and bending energy found a (local) minimum. We bring this idea back to computer graphics, defining physics-aware subdivision as a solution to quasi-static equations of motion based on elastic and bending energy of the surface.

Carrying on with our analogy to splines, we envision that for a given set of control points there exists one, or often many, plausible resultant smooth curves. Subdividing our coarse control polygon along the edges between control points we can grow each of these points out from the polygon surface in the normal direction. Each configuration of displacements has a potential energy. The natural tendency of cloth is to fold into a shape that minimizes the potential energy. We mimic this behaviour by optimizing over the space of displacements to find a local minimum of the potential energy and move to it at the end of every timestep. Though this method can't replace dynamic details missing from the coarse mesh, it does construct an explicit surface for rendering that reflects the implicit assumption made in the coarse mesh dynamics when an edge is compressed.

Our potential energy is a sum of the elastic or membrane energy, minimizing the strain across the deformable surface, and the bending energy, resisting sharp edges in folds. The local minimum of this potential energy results in a smooth, physically driven subdivision surface that preserves surface area as the coarse control mesh deforms.

Our system is easily combined with collision aware subdivision [BFA02] in order to handle collisions that occur as a result of the change in the displacements during the quasi-static timestep.



**Figure 2:** A set of 3 frames displaying the coarse (blue) and physics-aware subdivision (purple) meshes.

### 4. Results

Our system is able to successfully model the buckling and folding of a finely discretized mesh while keeping the motion true to the underlying coarse mesh. Figure 2 shows a few frames of a full 3-dimensional implementation, showing the coarse mesh which defines the motion and the fine mesh with physics aware subdivision applied. As triangles deform in the coarse mesh, folds begin to appear in the physically subdivided fine mesh.

### References

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