cs542g Final Exam
December 10, 2007

Attempt all questions. Partial marks will be awarded for demonstrating understanding of the relevant material even if you can't fully solve the problem.

1.....

The thin-plate spline radial basis function in 2D is $r^2 \log r$. This formula is undefined at $r = 0$; assuming the routine for computing $\log r$ has good relative accuracy, is it safe to rely on this formula for r very small but positive? What problem could you run across, and how might you fix it?

2.....

Prove how you can get the 2-norm condition number of a general square matrix from its SVD.

3.....

To cut down on the major expense of Newton's method for optimization, namely solving a linear system with the Hessian H, you could approximate H as a diagonal matrix $\bar{H}$ with $\bar{H}_{ii} = H_{ii}$ and thus make the solve trivial. How might this impact convergence?

4.....

The following algorithm is proposed for solving the second order problem $d^2x/dt^2 = a(x)$.

$$\begin{aligned} v_{n+1} &= v_n + \Delta t\, a(x_n) \\ x_{n+1} &= x_n + \Delta t \frac{v_n + v_{n+1}}{2} \end{aligned}$$

Is this an implicit or an explicit method? How accurate is it? How stable is it?

5.....

Consider taking a 2D regular grid, and solving the Poisson equation on it. Write down the simplest 2nd order accurate central finite difference scheme. If you split each grid square into two triangles by adding a diagonal, thus getting a triangle mesh, show that the stiffness matrix from Galerkin FEM with the usual continuous piecewise linear basis functions on that mesh is the same as the finite difference matrix up to a constant factor.