

1 Theory

(1) In class we considered several choices for Radial Basis Functions, such as $\phi(r) = r^2 \log r$, $\phi(r) = r$, or $\phi(r) = r^3$. What sort of results would you get with $\phi(r) = r^2$?

(2) Prove that the Moving Least Squares approximation, where it is well-defined, is as smooth as the weighting kernel used to define it. Hint: start by working out why this is true for a constant fit, then move on to a linear fit in 1D, and then try the general case. You need only report on the general case, but part marks will be awarded if you only manage one of these simpler cases.

2 Programming

(3) Write prototype RBF code in MATLAB for the 2D thin-plate spline case ($\phi(r) = r^2 \log r$), following the template provided in `solveRBF.m` and `evalRBF.m` (in the directory `matlabRBF`). The file `viewRBF.m` allows you to view your solution to check that it is reasonable.

(4) Write a compiled version of the same, using LAPACK to solve the linear system (routine `dsysv` for double precision floating point). Example C++ code is provided in the directory `cppRBF`.

(5) Implement (in a compiled language) the *ijk* and *kij* variants of the Cholesky factorization. Time those and LAPACK's `dposv` on a variety of problem sizes, and briefly comment on the differences. Example C++ code is provided in the directory `cppCholesky`.

You may find it helpful to write prototypes in MATLAB again, but timing information for interpreted MATLAB functions is of course irrelevant. Also note that Cholesky only applies to symmetric positive definite matrices, such as the normal equations in MLS, not the symmetric indefinite matrices involved in RBF interpolation.

3 Handing It In

Create a web-page for your solutions to this assignment, and email me the URL. You should include the solutions to theory questions 1 and 2; the code you write for questions 3, 4, and 5; example screenshots showing the results in questions 3 and 4; and a table of timings with comments for question 5. I expect the email to have arrived by the morning after the assignment is due.