Cloth Collisions/Contact

- Critical part of real-world clothing sims is collision
  - Not too many simple flags / curtains / table cloths in movies!
- This part of the course is concerned with making collisions
  1) good-looking,
  2) robust, and
  3) fast
  in that order
- References:
  - Provot, GI'97
  - Bridson, Fedkiw, & Anderson, SIGGRAPH'02
  - Bridson, Marino, & Fedkiw, SCA'03

Challenges

- Cloth is thin
  - Once you have a penetration, it's very obvious
  - Simulation might not be able to recover
- Cloth is flexible and needs many DOF
  - Dozens or hundreds of triangles, in many layers, can be involved simultaneously in collision area
- Cloth simulations are stressful
  - If something can break, it will…

Outline of Solution

- Separation from internal dynamics
- Repulsion forces
  - Well-conditioned, smooth, efficient for most situations
- Geometric collisions
  - Robust for high-velocity impacts
- Impact zones
  - Robust and scalable for highly constrained situations

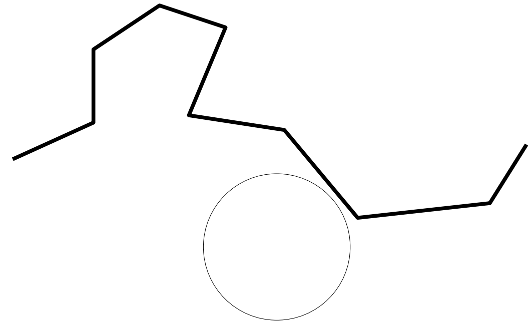
Separation from internal dynamics

Separation

- Simplify life by separating internal forces (stretching, bending) from collision forces
- Assume black-box internal dynamics:
  collision module is given
  1) x_0 at start of collision timestep, and
  2) x_1 candidate state at end
  and then returns
  3) x_{new} collision-free positions
- Time integrator responsible for incorporating this back into simulation

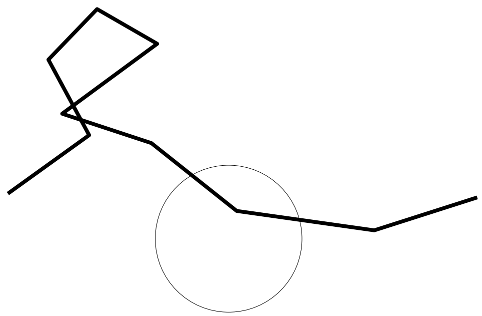
Example

- Start of timestep, x_0 (saved for collisions)



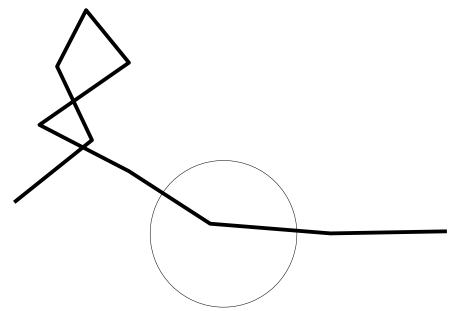
Example

- Take one or more internal dynamics steps (ignoring collisions)



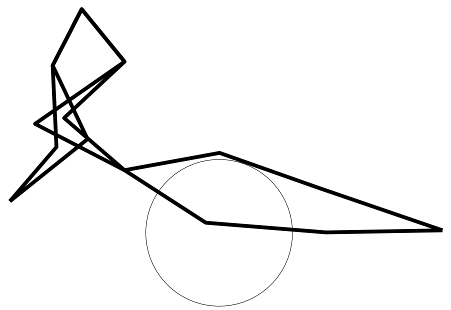
Example

- And get to x_1, candidate positions at end of collision step



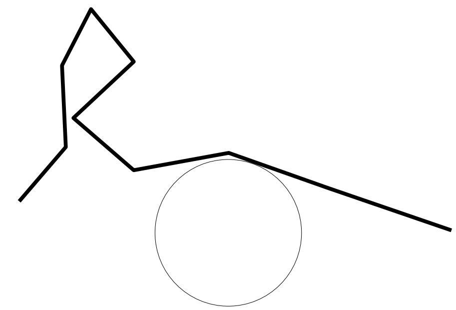
Example

- Looking at motion x_0 to x_1, collision module resolves collisions to get x_{new}



Example

- Time integrator takes x_{new} and incorporates collision impulses into velocity, continues on



Algorithm

- For n=0, 1, 2, …
  - (x, v) = advance_internal_dynamics(x_n, v_n, dt)
  - x_{n+1} = solve_collisions(x_n, x)
  - dv = $(x_{n+1} - x)/dt$
  - Optional:
    smooth dv with damping dynamics
    e.g. $dv = dv_{raw} + dt\ M^{-1}\ F_{damp}(x_{n+1}, dv)$
  - $v_{n+1} = v+dv$

Notes

- Collisions are synchronized, fixed time step is fine
- Cruder algorithm shown in [BFA'02]
- If elastic collisions are needed, add extra collision step using initial velocities v_n
  - see Guendelman, Bridson, Fedkiw, SIGGRAPH'03
- Solve_collisions() only needs x_0 and x_1:
  velocity is the difference $v=(x_1-x_0)$ when needed
- Assuming linear velocity dependence in velocity smoothing step
- Rest of talk: what to do in solve_collisions()

Repulsion Based Forces

Repulsions

- Look at old (collision-free) positions x_0
- If the cloth is too close to itself or something else, apply force to separate it
- Use these for modeling:
  - Cloth thickness (how close are repulsions active)
  - Cloth compressibility (how strong are they)
- Do not rely on them to stop all collisions
  - Extending influence and/or making them stiffer detracts from look of cloth, slows down simulation, …

Proximity Detection

- Two ways triangle meshes can be close:
  - Point close to triangle
  - Edge close to edge
- In both cases we will want to know barycentric coordinates of closest points

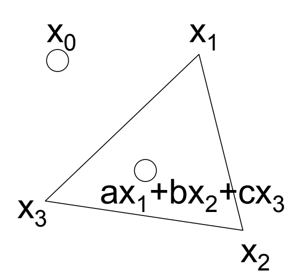
Point-Triangle Proximity

- Solve for barycentric coordinates of closest point on plane of triangle

$$\begin{pmatrix} |x_{13}|^2 & x_{13}\bullet x_{23} \\ x_{13}\bullet x_{23} & |x_{23}|^2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x_{13}\bullet x_{03} \\ x_{23}\bullet x_{03} \end{pmatrix}$$

$$c = 1 - a - b$$

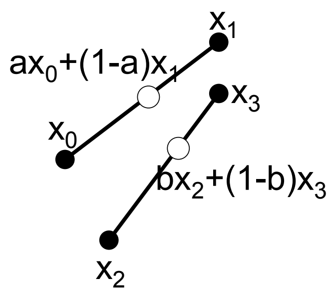
- If a barycentric coordinate is negative, skip this pair (edge-edge will pick it up)



Edge-Edge Proximity

- Solve for barycentric coordinates of closest points on infinite lines

$$\begin{pmatrix} |x_{01}|^2 & x_{01}\bullet x_{32} \\ x_{01}\bullet x_{32} & |x_{32}|^2 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} x_{01}\bullet x_{31} \\ x_{32}\bullet x_{31} \end{pmatrix}$$



- Clamp to finite segments - one that moved furthest is correct, project onto other line and clamp again for other point

Proximity Acceleration

- Put triangles in bounding volumes, only check elements if bounding volumes are close
- Organize bounding volumes for efficient culling
- Background grid works fine if triangles similar sizes
  - Check each element against others in its grid cell or nearby cells (within cloth thickness)
- Bounding volume hierarchies useful too
  - Prune checks between distant BV's and their children

BV Hierarchy Algorithm

- Put pair of root nodes on stack
- While stack not empty:
  - Pop the top pair of BV's
  - If they are close or overlapping:
    if leaves: check mesh elements
    else: push all pairs of children onto the stack

Computing Each Repulsion

- Direction of repulsion **n:**
  direction between closest points
  - In degenerate cases can use triangle normal or normalized edge cross-product
- Several choices for repulsion:
  - Damped spring between closest points, tries to pull them to cloth thickness apart
  - Kinematic result: move closest points some fraction of the way to cloth thickness apart

Finding the Impulse

- Example: point-triangle
  - Want to move closest points apart by distance **d**
  - Assume impulse distributed to corners of triangle by barycentric weights:

$$x_0^{new} = x_0 + \tfrac{1}{m_0} In \qquad x_1^{new} = x_1 - a\tfrac{1}{m_1} In$$
$$x_2^{new} = x_2 - b\tfrac{1}{m_2} In$$
$$x_3^{new} = x_3 - c\tfrac{1}{m_3} In$$

  - Then solve for impulse: (scripted nodes have ∞ mass)

$$\left[(x_0^{new} - ax_1^{new} - bx_2^{new} - cx_3^{new}) - (x_0 - ax_1 - bx_2 - cx_3)\right] \bullet n = d$$

$$\left(\frac{1}{m_0} + \frac{a^2}{m_1} + \frac{b^2}{m_2} + \frac{c^2}{m_3}\right) I = d$$

Friction

- Relative velocity: $v=(x_0^1-x_0^0)-a(x_1^1-x_1^0)-b(x_2^1-x_2^0)-c(x_3^1-x_3^0)$
- Tangential velocity: $v_T=v-(v\bullet n)n$
- Static: $v_T^{new}=0$ as long as $|F_T| < \mu F_N$
- Kinetic: If $v_T^{new}\neq 0$ then apply force $|F_T| = \mu F_N$
- Integrate forces in time: $F\rightarrow\Delta v$
- Combine into one formula:

Robustness Problem

- Repulsions only test for proximity at one time
- Fast moving cloth can collide in the middle of the time step, and repulsions won't see it
- Even if repulsions catch a fast collision, they may not resolve it
- End result: cloth goes through itself or objects
  - Once on the wrong side, repulsions will actively keep it there
  - Untangling is dodgy for self-intersections (but see Baraff et al, SIGGRAPH'03)

Robust Geometric Collisions

Collision Detection

- Not interference ("do the meshes intersect?"), but true collision detection ("do the trajectories hit at any intermediate time?")
- Again: meshes can collide in two ways
  - Point hits triangle, edge hits edge
- Approach (Provot'97):
  - Assume constant velocity of nodes through timestep
  - Solve for times when nodes coplanar (cubic in t)
  - Check proximity (some tolerance) at possible collision times

Defining the Cubic

- Assume $x_i(t) = x_i + t\, v_i$ (with $0 \le t \le 1$)
- Coplanar when tetrahedral volume of (x_0, x_1, x_2, x_3) is zero, i.e. when

$$\left[x_1(t) - x_0(t),\ x_1(t) - x_0(t),\ x_1(t) - x_0(t)\right] = 0$$

- Reduces to a cubic in t:

$$\left[v_{10}, v_{20}, v_{30}\right]t^3 + \left(\left[x_{10}, v_{20}, v_{30}\right] + \left[v_{10}, x_{20}, v_{30}\right] + \left[v_{10}, v_{20}, x_{30}\right]\right)t^2$$
$$+\left(\left[x_{10}, x_{20}, v_{30}\right] + \left[x_{10}, v_{20}, x_{30}\right] + \left[v_{10}, x_{20}, x_{30}\right]\right)t + \left[x_{10}, x_{20}, x_{30}\right] = 0$$

Solving the Cubic

- We can't afford to miss any collisions: have to deal with floating-point error
  - Closed form solution not so useful
- Take a root-finding approach:
  - Solve derivative (quadratic) for critical points
  - Find subintervals of [0,1] where there could be roots
  - Find roots in each subinterval with a sign change using secant method
  - If cubic evaluates close enough to zero at any point (e.g. subinterval boundaries), count as a root -- even with no sign change

Acceleration

- Extend bounding volumes to include entire trajectory of triangle
- Then acceleration is exactly the same as for proximity detection

Collision Impulse

- Use the normal of the triangle, or normalized cross-product of the edges, at collision time
- Inelastic collisions assumed:
  want relative normal velocity to be zero afterwards
- Solve for impulse exactly as with repulsions
- Friction (tangential velocity modification) also works exactly the same way

Iteration

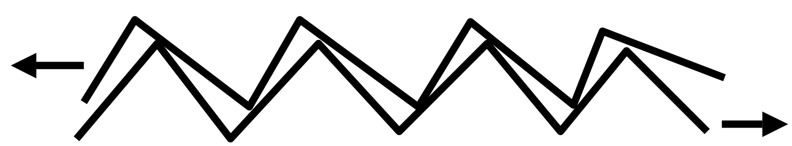
- Each time we collide a pair, we modify their end-of-step positions
- This changes trajectories of coupled elements: could cause new collisions
- So generate the list of potentially colliding pairs, process them one at a time updating x_{new} as we go
- Then generate a new list -- keep iterating

1) Scalability Problem

- Resolving one pair of colliding elements can cause a coupled pair to collide
  - Resolving that can cause the first to collide again
    - Resolving the first ones again can cause others to collide
      - And so on…
- Easy to find constrained situations which require an arbitrary number of iterations

2) Modeling Problem

- Chainmail friction: wrinkles stick too much
  - Triangles behave like rigid plates,
    must be rotated to slide over each other,
    takes too much torque



3) Robustness Problem

- Cloth can get closer and closer,
  until…
  floating-point error means we're not sure which side things are on
- To be safe we need cloth to stay reasonably well separated

Impact Zones

Attack Scalability Issue

- Pairwise impulses are too local: need global resolution method
  - [Provot'97, BFA'02]: rigid impact zones
- Note: a set of intersection-free triangles remain intersection-free during rigid motion
- So when a set of elements ("impact zone") collides, rigidify their motion to resolve

Impact Zones

- Initially each vertex is its own impact zone
- Look for point-triangle and edge-edge collisions between distinct impact zones:
  - Merge all involved impact zones (at least 4 vertices) into a single impact zone
  - Rigidify motion of new impact zone
- Keep looking until everything is resolved

Rigidifying

- Need to conserve total linear and angular momentum of impact zone:
  - Compute centre of mass
  - Compute linear and angular momentum
  - Compute total mass and inertia tensor of vertices
  - Solve for velocity and angular velocity
  - Compute each new vertex position from translation+rotation
- Treat any scripted vertices as having ∞ mass
- Note: if impact zone spans non-rigid scripted vertices, you're in trouble…. try cutting the timestep

1) Damping Problem

- Rigidifying eliminates more relative motion than desired: infinite friction
- Could see rigid clumps in motion

2) Robustness Problem

- Just like pair-wise impulses, cloth may get closer and closer in simulation
- At some point, floating-point error causes collision detection to break down
- Impact zones will never separate then

Putting it Together

Three Methods

- Repulsions
  ☺ cheap, well behaved
  ☹ not robust
- Collisions
  ☺ can catch high velocity events
  ☹ not scalable in constrained situations
  ☹ “chainmail” false friction
  ☹ robustness problem when cloth gets too close
- Impact Zones
  ☺ scalably robust
  ☹ over-damped dynamics
  ☹ robustness problem when cloth gets too close

Pipeline

- First use repulsions
  - Handles almost all interaction (contact mostly)
  - Keeps cloth nicely separated
  - Models thickness and compressibility
- Then run geometric collisions on that output
  - Catches isolated high velocity events accurately
- Finally use impact zones as a last resort
  - In the rare case something remains
- Note: repulsions make it all work well