Notes

- Please read
  - O'Brien and Hodgins, "Graphical modeling and animation of brittle fracture", SIGGRAPH '99
  - O'Brien, Bargteil and Hodgins, "Graphical modeling and animation of ductile fracture", SIGGRAPH '02, pp. 291--294.

Discrete Mean Curvature

- [draw triangle pair]
- κ for that chunk varies as
- So integral of κ^2 varies as

$$\kappa \sim \frac{\theta}{h_1 + h_2}$$

$$W = \sum_e \frac{\theta^2}{(h_1 + h_2)^2}\left(|\Delta_1| + |\Delta_2|\right) \sim \sum_e \frac{\theta^2 |e|^2}{|\Delta_1| + |\Delta_2|}$$

- Edge length, triangle areas, normals are all easy to calculate
- θ needs inverse trig functions
- But θ^2 behaves a lot like 1-cos(θ/2) over interval [$-\pi$,π] [draw picture]

Bending Force

- Force on x_i due to bending element involving i is then

$$F_i = -B\frac{\partial W}{\partial x_i} \sim -B\frac{|e|^2}{|\Delta_1| + |\Delta_2|}\sin\left(\frac{\theta}{2}\right)\frac{\partial \theta}{\partial x_i}$$

- Treat first terms as a constant (precompute in the rest configuration)

$$\sin\tfrac{\theta}{2} = \pm\sqrt{\tfrac{1}{2}\left(1 - n_1 \cdot n_2\right)}$$

- Sign should be the same as $\sin\theta = n_1 \times n_2 \cdot \hat{e}$
- Still need to compute $\partial\theta/\partial x_i$

Gradient of Theta

- Can use implicit differentiation on cos(theta)=$n_1 \cdot n_2$
  - Not too much fun
  - Automatic differentiation: Grinspun et al. "Discrete Shells, SCAN'03
  - Modal analysis: Bridson et al., "Simulation of clothing…", SCA'03

Damping hyper-elasticity

- Suppose W is of the form C• C / 2
  - C is a vector or function that is zero at undeformed state
- Then F = -∂C/∂X • C
  - C says how much force, ∂C/∂X gives the direction
- Damping should be in the same direction, and proportional to ∂C/∂t:
  F = -∂C/∂X • ∂C/∂t
- Can simplify with chain rule:
  F = -∂C/∂X • (∂C/∂X v)
  - Linear in v, but not in x…

Hacking in strain limits

- Especially useful for cloth:
  - Biphasic nature: won't easily extend past a certain point
- Sweep through elements (e.g. springs)
  - If strain is beyond given limit, apply force to return it to closest limit
  - Also damp out strain rate to zero
- No stability limit for fairly stiff behaviour
- See X. Provot, "Deformation constraints in a mass-spring model to describe rigid cloth behavior", Graphics Interface '95

Elastic Collisions

Simplest approach

- Treat it just as a particle system:
  - Check if (surface) particles hit objects
  - Process collisions independently if so
- Inelastic collisions (and simplified resolution algorithm) are perfectly appropriate
  - Elasticity/damping inside object itself provides the rebound…
- Problems:
  - Coupling with uncollided particles?
  - Thin objects (like cloth or hair)?
  - Deformable vs. deformable?

Coupling with rest of object

- Velocity smoothing:
  - Figure out collision velocity, call it $v_{n+1/2}$
    $x_{n+1}=x_n+\Delta t\ v_{n+1/2}$
  - Then do an implicit velocity update:
    $v_{n+1}=v_{n+1/2} + \Delta t/2\ a(x_{n+1},v_{n+1})$
  - See Bridson et al., "Robust treatment of collisions…", SIGGRAPH '02
- Stronger velocity smoothing
  - Constrain normal velocity of colliding nodes
  - See Irving et al., "Invertible finite elements…", SCA '04
- Couple into full implicit solve (position as well)
  - See Baraff & Witkin, "Large steps in cloth simulation", SIGGRAPH '98

Thin objects

- Collision detection is essential
  - Otherwise particles will jump through objects
- But not enough
  - Triangle mesh vs. mesh requires point vs. face AND edge vs. edge
  - Otherwise we can have significant tangling

Distributing impulses

- If an edge collides, how do we distribute impulse between two endpoints?
- If a triangle collides, how do we distribute impulse between three corners?
- Weight with barycentric coordinates
  - And require that interpolated point change in velocity is what is required
- See Bridson et al., "Robust treatment…", SIGGRAPH '02

Scalable collision processing

- Cloth: fixing one collision can cause others
  - Easy to find situations where 1000+ iterations required
- Rigid impact zones: X. Provot, "Collision and self-collision handling in cloth model dedicated to design garment" Graphics Interface 1997
  - And Bridson Ph.D. thesis 2003
- When two regions collide, merge region and project velocities onto rigid or affine motions
- Efficiently resolves everything (but overdamped)
- Use as the last resort

Additional repulsions

- Avoid alligator teeth problem - triangle locking - with three steps:
  - Apply soft repulsion forces (at level comparable to geometry approximation)
  - Detect collisions, apply impulses
  - Rigid impact zones

Plasticity & Fracture

Plasticity & Fracture

- If material deforms too much, becomes permanently deformed: plasticity
  - Yield condition: when permanent deformation starts happening (“if stress is large enough”)
  - Elastic strain: deformation that can disappear in the absence of applied force
  - Plastic strain: permanent deformation accumulated since initial state
  - Total strain: total deformation since initial state
  - Plastic flow: when yield condition is met, how elastic strain is converted into plastic strain
- Fracture: if material deforms too much, breaks
  - Fracture condition: “if stress is large enough”

For springs (1D)

- Go back to Terzopoulos and Fleischer
- Plasticity: change the rest length if the stress (tension) is too high
  - Maybe different yielding for compression and tension
  - Work hardening: make the yield condition more stringent as material plastically flows
  - Creep: let rest length settle towards current length at a given rate
- Fracture: break the spring if the stress is too high
  - Without plasticity: “brittle”
  - With plasticity first: “ductile”