1 Implementation

In this assignment you will simulate 3D smoke, with an exact projection algorithm on a staggered grid, using semi-Lagrangian advection (Forward Euler tracing, limited cubic Hermite interpolation). We'll simplify from class a little by assuming that smoke concentration and temperature are proportional, so the buoyancy force is just $f_b = Bs$ where B is a constant given as a parameter and s is the smoke concentration.

The domain will be a specified cube of dimensions $nx \times ny \times nz$, where a grid cell is of dimension 1×1 (without viscosity, there is no inherent length scale to the problem, so we're not going to worry about keeping track of Δx's). The boundary conditions are open (u and s not specified, $p = 0$) on all sides except the ground $y = 0$, where v and s are specified.

The initial conditions are $u = 0$ and $s = 0$ everywhere except on the ground plane.

Part of the parameters to the simulation is a specification of what is happenning on the ground plane. There the smoke concentration s is given, and the v component of velocity is taken to be $3Bs$. So smoke should start pouring out of the ground.

Also in the parameters is how many seconds to run the simulation for (writing frames 24 times a second; we can take one timestep equal to one frame), and how many iterations to take in the pressure solver.

Instead of PCG, we'll use a much simpler (and slower) solver called Gauss-Seidel. That is, each iteration loop through the grid updating p by

$$p_{ijk} \leftarrow \frac{b_{ijk} + p_{i+1jk} + p_{i-1jk} + p_{ij+1k} + p_{ij-1k} + p_{ijk+1} + p_{ijk-1}}{A_{ijk,ijk}}$$

sticking in zeros for values of pressure off the grid, and where $A_{ijk,ijk}$ is 6 for most grid nodes, but 5 for the ground ($j = 0$) due to the closed boundary condition there. The right-hand side b comes from the divergence of the velocity field.

You will find example code provided, as well as an OpenGL-based viewer.

2 Analysis

For the 1D advection equation $q_t = -uq_x$ where $u > 0$ is a constant, write down the 1st order upwind scheme with Forward Euler in time, and write down the semi-Lagrangian scheme with Forward Euler tracing (which is exact in this case) and linear interpolation. Show they are equal for some time steps Δt, and tell me which time steps this is the case. What happens to the 1st order upwind scheme when Δt is larger than that?

3 Feedback

Please grade my assignments – which ones you thought were too hard, which ones were too easy, which ones needed more code provided or more time, which ones didn't teach you very much, etc.

4 Project Ideas

You can extend this assignment in a number of ways for your final project, for example:

- add vorticity confinement
- improve the advection to use better tracing, and add an object in the domain that the smoke will flow around
- replace the linear solver with PCG (using an appropriate preconditioner) and compare the difference in speed

- visualize the smoke by tracing a particle system through the flow
- turn this into a water simulation (a little challenging)