Notes

- Typo in blending function for homework 6 viewer
  - Thanks to Albert Wong for pointing it out
- We’ve almost finished surveying physics-based animation
  - If you want me to review anything, get into more detail anywhere, talk about anything more exotic, let me know!

Fire

- [Nguyen, Fedkiw, Jensen ‘02]
- Gaseous fuel/air mix (from a burner, or a hot piece of wood, or ...) heats up
- When it reaches ignition temperature, starts to burn
  - “blue core” - see the actual flame front due to emission lines of excited hydrocarbons
- Gets really hot while burning - glows orange from blackbody radiation of smoke/soot
- Cools due to radiation, mixing
  - Left with regular smoke

Defining the flow

- Inside and outside blue core, regular incompressible flow with buoyancy
- But an interesting boundary condition at the flame front
  - Gaseous fuel and air chemically reacts to produce a different gas with a different density
  - Mass is conserved, so volume has to change
  - Gas instantly expands at the flame front
- And the flame front is moving too
  - At the speed of the flow plus the reaction speed

Interface speed

- Interface = flame front = blue core surface
- \( D = V_f - S \) is the speed of the flame front
  - It moves with the fuel flow, and on top of that, moves according to reaction speed \( S \)
  - \( S \) is fixed for a given fuel mix
- We can track the flame front with a level set \( \phi \)
- Level set moves by
  \[
  \frac{\partial \phi}{\partial t} + D \nabla \phi = 0
  \]
  \[
  \phi_t + u_{LS} \cdot \nabla \phi = 0
  \]
- Here \( u_{LS} \) is \( u_f - Sn \)
**Numerical method**

- For water we had to work hard to move interface accurately.
- Here it’s ok just to use semi-Lagrangian method (with reinitialization).
- Why?
  - We’re not conserving volume of blue core - if reaction is a little too fast or slow, that’s fine.
  - Numerical error looks like mean curvature.
  - Real physics actually says reaction speed varies with mean curvature.

**Conservation of mass**

- Mass per unit area entering flame front is $\rho_f(V_f - D)$ where
  - $V_f = u_f \cdot n$ is the normal component of fuel velocity.
  - $D$ is the (normal) speed of the interface.
- Mass per unit area leaving flame front is $\rho_h(V_h - D)$ where
  - $V_h = u_h \cdot n$ is the normal component of hot gaseous products velocity.
- Equating the two gives:
  $$\rho_f(V_f - D) = \rho_h(V_h - D)$$

**Velocity jump**

- Plugging interface speed $D$ into conservation of mass at the flame front gives:
  $$\rho_f S = \rho_h \left(V_h - V_f + S\right)$$
  $$\rho_h V_h = \rho_h V_f + \rho_f S - \rho_h S$$
  $$V_h = V_f + \left(\frac{\rho_f}{\rho_h} - 1\right) S$$

**Ghost velocities**

- This is a “jump condition”: how the normal component of velocity jumps when you go over the flame interface.
- This lets us define a “ghost” velocity field that is continuous.
  - When we want to get a reasonable value of $u_h$ for semi-Lagrangian advection of hot gaseous products on the fuel side of the interface, or vice versa (and also for moving interface).
  - When we compute divergence of velocity field.
- Simply take the velocity field, add/subtract $(\rho_f/\rho_h - 1) S n$.
Conservation of momentum

- Momentum is also conserved at the interface.
- Fuel momentum per unit area "entering" the interface is
  \[ \rho_f V_f (V_f - D) + p_f \]
- Hot gaseous product momentum per unit area "leaving" the interface is
  \[ \rho_h V_h (V_h - D) + p_h \]
- Equating the two gives
  \[ \rho_f V_f (V_f - D) + p_f = \rho_h V_h (V_h - D) + p_h \]

Simplifying

- Make the equation look nicer by taking conservation of mass:
  \[ \rho_f (V_f - D) = \rho_h (V_h - D) \]
  multiplying both sides by -D:
  \[ \rho_f (-D)(V_f - D) = \rho_h (-D)(V_h - D) \]
  and adding to previous slide’s equation:
  \[ \rho_f (V_f - D)^2 + p_f = \rho_h (V_h - D)^2 + p_h \]

Pressure jump

- This gives us jump in pressure from one side of the interface to the other.
- By adding/subtracting the jump, we can get a reasonable continuous extension of pressure from one side to the other.
  - For taking the gradient of \( p \) to make the flow incompressible after advection.
- Note when we solve the Poisson equation density is NOT constant, and we have to incorporate jump in \( p \) (known) just like we use it in the pressure gradient.

Temperature

- We don’t want to get into complex (!) chemistry of combustion.
- Instead just specify a time curve for the temperature:
  - Temperature known at flame front \( (T_{ignition}) \)
  - Temperature of a chunk of hot gaseous product rises at a given rate to \( T_{max} \) after it’s created.
  - Then cools due to radiation.
Temperature cont’d

• For small flames (e.g. candles) can model initial temperature rise by tracking time since reaction: $Y_t + u \cdot \nabla Y = 1$ and making $T$ a function of $Y$
• For large flames ignore rise, just start flame at $T_{max}$ (since transition region is very thin, close to blue core)
• Radiative cooling afterwards:

$$T_t + u \cdot \nabla T = -c_T \left( \frac{T - T_{air}}{T_{max} - T_{air}} \right)^4$$

Smoke concentration

• Can do the same as for temperature: initially make it a function of time $Y$ since reaction (rising from zero)
  • And ignore this regime for large flames
  • Then just advect without change, like before
• Note: both temperature and smoke concentration play back into velocity equation (buoyancy force)

Note on fuel

• We assumed fuel mix is magically being injected into scene
  • Just fine for e.g. gas burners
  • Reasonable for slow-burning stuff (like thick wood)
• What about fast-burning material?
  • Can specify another reaction speed $S_{fuel}$ for how fast solid/liquid fuel turned into flammable gas (dependent on temperature)
  • Track level set of solid/liquid fuel just like we did the blue core