Notes

- Next week:
  - Guest lecture on Tuesday
  - Your project presentations on Thursday
  - Make sure you can produce animations for then (if you have problems, let me know ASAP)
  - If you're not using MPEG or QuickTime, send me a sample animation in your chosen format to make sure I can display it in class!
- Teaching evaluations today

Clouds

- Another extension of incompressible flow
- Basic physics:
  - Humid air rises (due to buoyancy)
  - As altitude increases, temperature and pressure decrease
  - Sooner or later (started by dust or other particles) water has to condense
  - Very tiny water droplets form (radius about 10μm), hundreds per cubic cm --- this is the visible cloud
    - (and rain, snow, hail etc. form when droplets get bigger, start to merge, freeze, etc.)
  - Condensation releases heat, causes more buoyancy

Basis

- [Harris '03] "Simulation of cloud dynamics on graphics hardware"
- Need to track
  - Velocity u
  - Potential temperature θ
  - Mass fraction of water vapour q_v
  - Mass fraction of condensed water q_c
- The new variables feed into buoyancy force for velocity
- Advection + thermodynamics lets us solve for the new variables

Heat and temperature

- Heat (energy) is fundamental
  - Absolute temperature also depends on pressure, heat capacities, etc.
- When background conditions are basically constant, heat and temperature are proportional
- In the atmosphere, they're not
- So we should use something more directly related to heat than absolute temperature

Potential Temperature

- Density, pressure and temperature vary significantly with altitude
- We're only looking at small variations from background values
  - so Boussinesq approximation is valid
- θ is temperature resulting from the same heat in the (dry) air, but at sea-level pressure
  - "adiabatically" moved down to sea-level
  - Ignoring small variations from background values
  - So θ is more of a measure of heat, but in units of temperature

Buoyancy

- Again, just buoyancy force is just the variation in force due to gravity resulting from variation in density

$$f_{buoyancy} = \frac{\rho_{background} + \Delta\rho}{\rho_{background}} g$$

- Adding or subtracting a constant body force (e.g. g) is compensated by linear pressure gradient
- So we can cancel that part out, left with

$$f_{buoyancy} = \frac{\Delta\rho}{\rho_{background}} g$$

Calculating buoyancy

- Relative density changes due to thermal expansion
  - Also add in effect of water vapour in the air - changes heat capacity and density, so a given potential temperature results in a different thermal expansion
- Presence of condensed water also increases density

$$g\left(\frac{\theta(1+0.61q_v)}{\theta_{v0}} - q_c\right)$$

Condensation

- q_v and q_c advect around, and convert from one to the other (q_v+q_c conserved)
- Rate of condensation: $\frac{Dq_c}{Dt} = C, \quad \frac{Dq_v}{Dt} = -C$
- Very simple model of condensation:
  - Water evaporates instantly up to the saturation point, any excess water condenses
  - Compute C to make sure this holds
- Critical value is saturation point q_{vs}
  - Depends on pressure, absolute temperature, …

Saturation point

- Empirical model: $$q_{vs} = \frac{380.16}{p} \exp\left(\frac{17.67\,T}{T + 243.5}\right)$$
- Background pressure p is $$p = p_0\left(1 - \frac{y\Gamma}{T_0}\right)^{\frac{g}{\Gamma R_{dry}}}$$
  - With pressure $p_0 \approx 101$Kpa and temperature $T_0 \approx 300$K (in Kelvins) at sea-level, $\Gamma \approx 10$K/km is the "lapse rate" or how fast temperature drops with altitude near Earth, and $R_{dry} \approx 287$J/(kg K) is the gas constant for dry air
- Temperature T is in Celsius…

Saturation point cont'd

- Temperature T_K in Kelvin (from absolute zero) is based on potential temperature:

$$T_K = \Pi\theta$$

$$= \left(\frac{p}{p_0}\right)^{\kappa}\theta = \left(\frac{p}{p_0}\right)^{\frac{R_{dry}}{c_p}}\theta \approx \left(\frac{p}{p_0}\right)^{0.286}\theta$$

  - Π is the "Exner function", $c_p \approx 1000$J/(kg K) is specific heat of dry air at constant volume
  - For saturation point, use T_0 for θ
- Subtract 287K to get T in Celsius

Latent heat

- One last effect
  - As water condenses, it releases latent heat
  - As water evaporates, it takes heat
  - (How your fridge works)
- So potential temperature is advected around, and changes according to rate of condensation: $$\frac{D\theta}{Dt} = \frac{L}{c_p\Pi}C$$
- Here $L \approx 2.5$J/kg is latent heat

Putting it together

- Advect quantities around
- Add buoyancy force and others (vorticity confinement)
- Calculate condensation amount $C\Delta t = \max(q_v - q_{vs}, -q_c)$
- Update q_v, q_c and θ from ΔC
- Do pressure solve to enforce incompressibility

What next?

- Other phenomena
  - Geophysical flows (hurricanes?)
  - Freezing, melting, boiling, …
  - Multiphysics
- More subtle physics effects
  - Avoid some of the simplifying assumptions
  - Higher accuracy numerical methods
- Fighting resolution
  - For 3D we scale like $O(n^3)$ at best, often worse
  - Adaptive methods, clever hacks
- Control, user-interface

Final notes

- Marking - yes, I'm behind
- I will try to get higher quality software renderers up on the web (for the assignments) as soon as I can
- I'll post movies of the assignments on the web