Relational Probabilistic Models

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Work with: http://minervaintelligence.com, https://treatment.com/

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Outline

1. Knowledge Graphs
   - Triples and Reification
   - Tensor Factorization and Neural Network Models
2. Representation Issues
   - Desiderata
   - How do relational models relate to probabilistic graphical models
3. Unique properties of relational models
   - Learning general knowledge vs learning about a data set
   - Varying Populations
   - What can be observed?
4. Conclusions and Challenges

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With a single relation it can be implicit $\longrightarrow$ triples:
$\langle pen_7, color, red \rangle$.

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- $prop(a, type, parcel)$, where *type* is a special property and *parcel* is a class.
- $prop(a, parcel, true)$, where *parcel* is a Boolean property (characteristic function of the class *parcel*).

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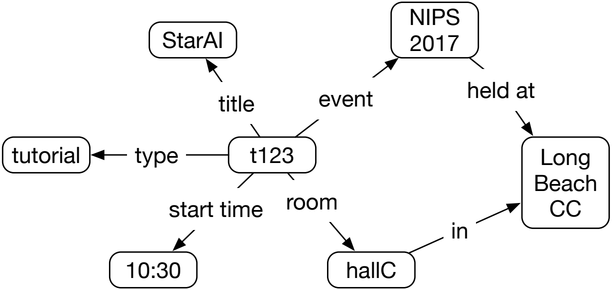
- How can we add extra arguments (e.g., presenters, chair)?

Triples and Knowledge Graphs

- Subject–verb–object
  Individual–property–value
  triples can be depicted as edges in graphs



- A modeller or learner needs to invent (reified) objects.

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All relations can be represented in terms of triples:

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prop(*Individual*, *Property*, *Value*) is the only relation needed: $\langle Individual, Property, Value \rangle$ triples, Semantic network, entity relationship model, knowledge graphs, ...

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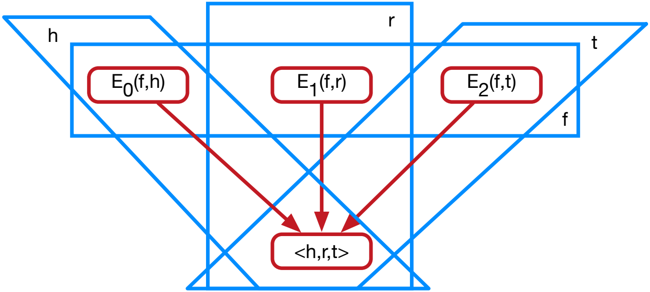

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- CompleX: the embeddings are complex numbers, tail is the conjugate of the head embedding
- SimpleE: have an embedding for r^{-1} and learn to predict both $\langle h, r, t\rangle$ and $\left\langle t, r^{-1}, h\right\rangle$

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- Which is better depends on the goals and how success is measured.
- Many of the methods try to do both; learn about specific individuals and general knowledge.

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- Difficult to learn about reified entities.

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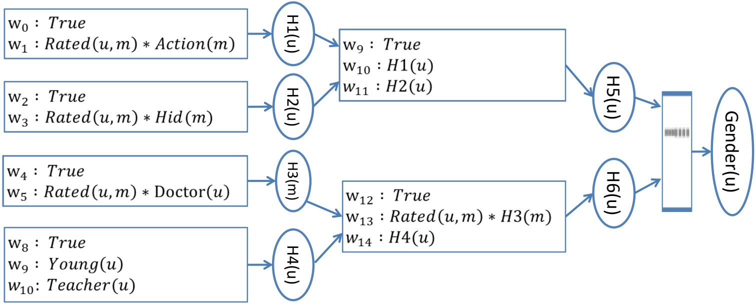
- We can build relational neural networks to solve this

[Kazemi & Poole, AAAI 2017]

Relational Neural Nets (RelNNs)



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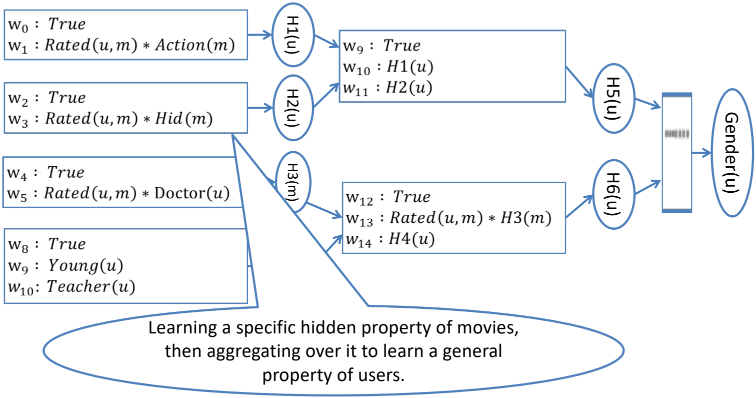

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- Modularity:
  Can independently developed parts be combined to form larger model?
  Can a larger model be decomposed into smaller parts?

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- Directed (and undirected) models are universal: can represent any probability distribution over a finite set of variables
- Algorithms developed for undirected models work for both.
- That does not mean that representations for undirected models can represent directed models.

Example

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$$P(\mathit{funFor}(X) \mid \Pi) = \mathit{sigmoid}(-5 + 10 n_T)$$

n_T is the number of individuals Y for which $\mathit{knows}(X, Y) \land \mathit{social}(Y)$ is *True* in Π.

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- Using wighted formulae to define conditional probabilities $\rightarrow$ relational logistic regression (RLR).
- Using wighted formulae to define distributions $\rightarrow$ Markov logic networks (MLNs).

Abstract Example

$\alpha_0 : q$
$\alpha_1 : q \wedge \neg r(x)$
$\alpha_2 : q \wedge r(x)$
$\alpha_3 : r(x)$

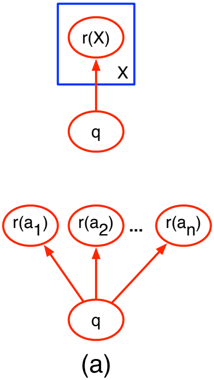
If $r(x)$ for every individual x is observed:

$$P(q \mid obs) = sigmoid(\alpha_0 + n_F \alpha_1 + n_T \alpha_2)$$

n_T is number of individuals for which $r(x)$ is true
n_F is number of individuals for which $r(x)$ is false

$$sigmoid(x) = \frac{1}{1 + e^{-x}}$$

Three Elementary Models



(a)

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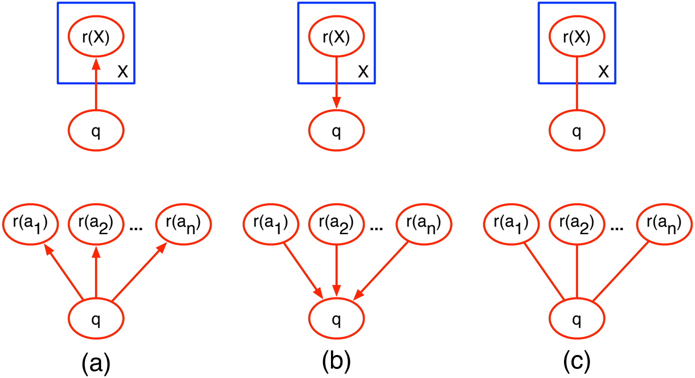

(a) Naïve Bayes
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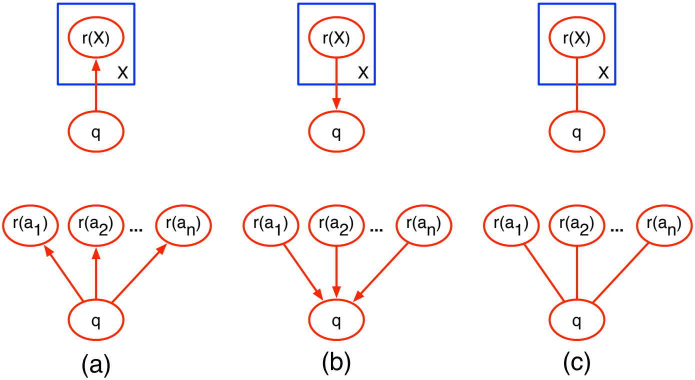

(a) Naïve Bayes

(b) (Relational) Logistic Regression

(c)

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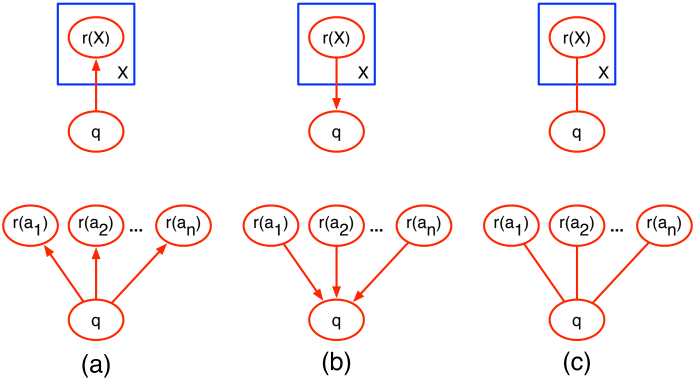

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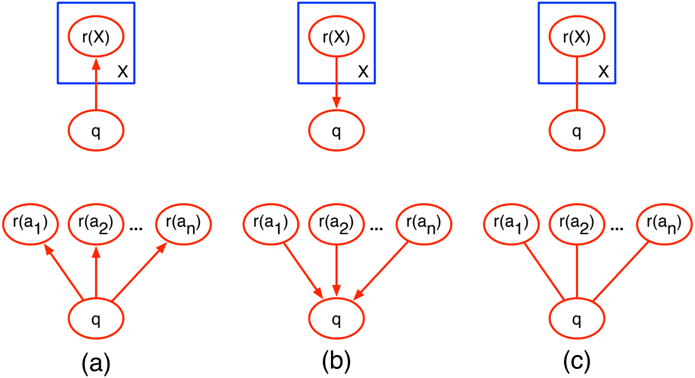

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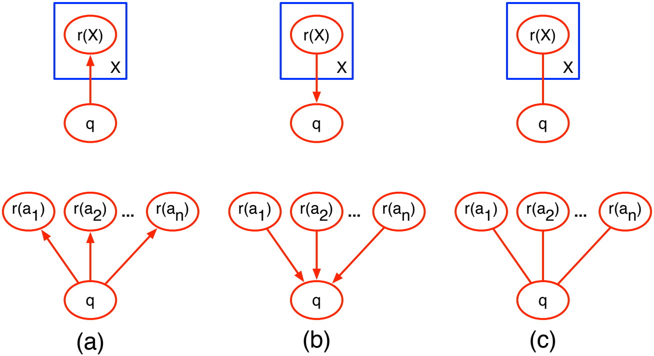
— They are identical models when all r's are observed.

Independence Assumptions



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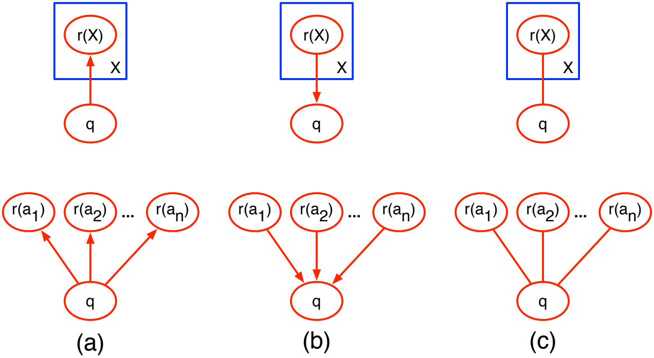

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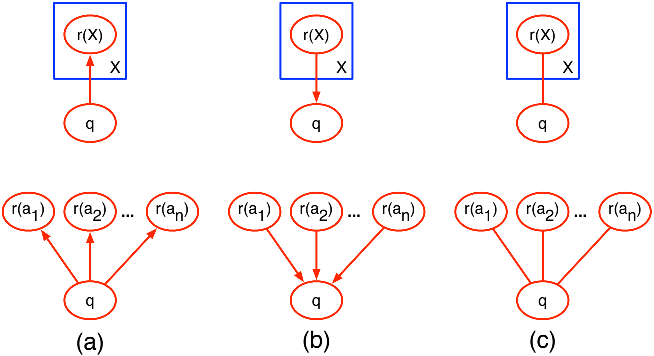

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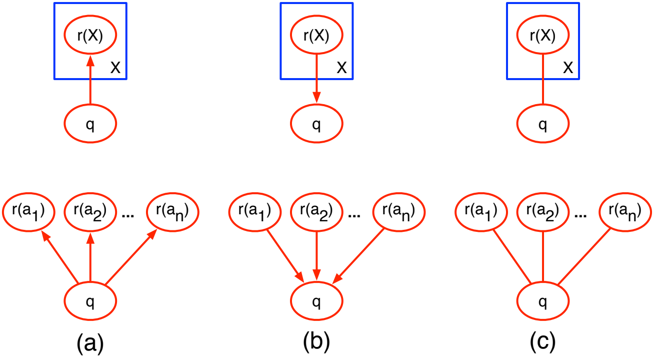

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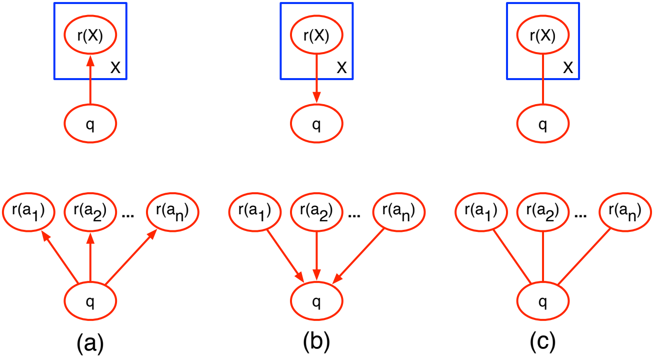

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- Directed models are inherently modular.
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  - if q is summed out, the distribution over $r(a_1) \ldots r(a_n)$ is not changed.
  - Why? requires factors on arbitrary subsets of $r(a_1) \ldots r(a_k)$
    — pairwise (or 3-wise or ...) interactions are not adequate
    — can't marry the parents

Cyclic Models

Whether people smoke depends on whether their friends smoke.

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$$w : smokes(X) \leftarrow friends(X, Y) \wedge smokes(Y)$$

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  - Probability of smokes goes up as the number of friends increases!
  - ICL/Problog cannot represent negative effects: someone is less likely to smoke if their friends smoke (without there being a non-zero probability of logical inconsistency)

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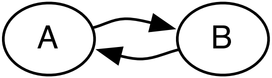
  Destroys exchangeability. Symmetries are not preserved.
- (Relational) dependency networks: directed model,



  - $P(A, B)$ has 3 degrees of freedom,
  - $P(A \mid B), P(B \mid A)$, uses 4 numbers; typically inconsistent.
  - resulting distribution means stationary (equilibrium) distribution of Markov chain.

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- Which is better depends on the goals and how success is measured.
- Many of the methods try to do both; learn about specific individuals and general knowledge.

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- Tensor factorization models — which learn vectors for individuals — tend to not learn generalized knowledge but about the particular individuals
- Lifted graphical models (MLNs, RLR, Problog) learn general knowledge through training weights (and structure) and specific knowledge through conditioning.
- Still open research problem

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What happens as Population size n Changes: Simplest case

$$\begin{aligned}
\alpha_0 &: q \\
\alpha_1 &: q \wedge \neg r(x) \\
\alpha_2 &: q \wedge r(x) \\
\alpha_3 &: r(x)
\end{aligned}$$

Weighted formulae define distribution:

$$P_{MLN}(q \mid n) = sigmoid(\ \alpha_0 + n \log(e^{\alpha_2} + e^{\alpha_1 - \alpha_3})\ )$$

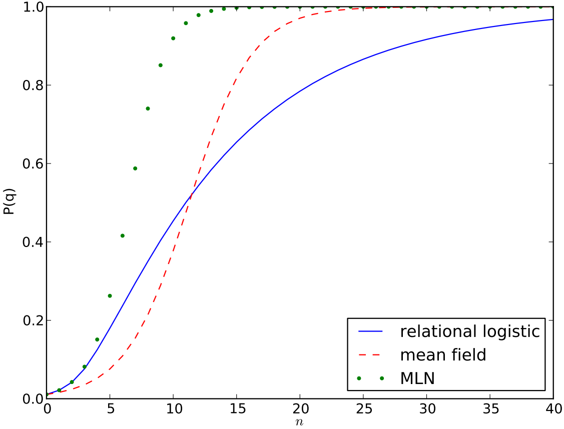
Weighted formulae define conditionals:

$$P_{RLR}(q \mid n) = \sum_{i=0}^{n} \binom{n}{i} sigmoid(\alpha_0 + i\alpha_1 + (n-i)\alpha_2)(1-p_r)^i p_r^{n-i}$$

Mean-field approximation:

$$P_{MF}(q \mid n) = sigmoid(\alpha_0 + np_r\alpha_1 + n(1-p_r)\alpha_2)$$

Population Growth: $P(q \mid n)$



All: $P(q \mid n) \rightarrow$ 0 or 1 as $n \rightarrow \infty$

Challenges of varying populations

- Example: The Movielens 100k dataset contains data about $rated(P, M, R, T)$ meaning person P gave movie M a rating of R at time T.
  Plus user demographic and movie information.
- Number of ratings per user is between 20 (arbitrary threshold) and 737; average of 106
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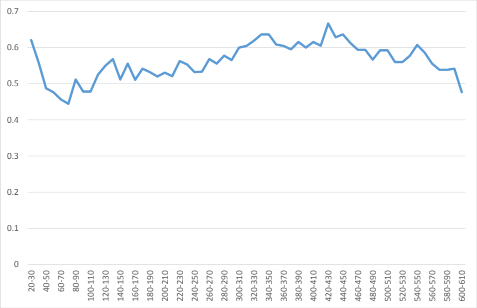
- Bigger datasets have even more variability.

Real Data



Observed $P(25 < Age(u) < 45 \mid n)$, where n is number of movies watched from the Movielens dataset.

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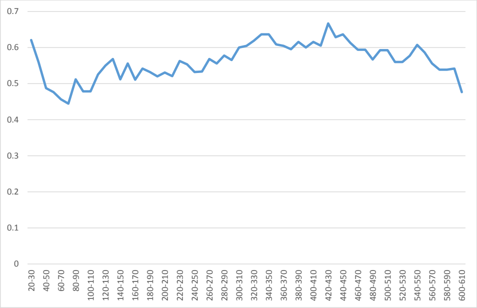

Observed $P(25 < Age(u) < 45 \mid n)$, where n is number of movies watched from the Movielens dataset.
Dont use:

$$w : middle\_age(U) \leftarrow rated(U, M) \land foo(M)$$

then $P(middle\_age(U)) \rightarrow$ 0 or 1 as number of movies increases.

Example of polynomial dependence of population

$\alpha_0 : q$

$\alpha_1 : q \wedge true(X)$

$\alpha_2 : q \wedge r(X)$

$\alpha_3 : true(X)$

$\alpha_4 : r(X)$

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In RLR and in MLN, if all $r(a_i)$ are observed:

$$P(q \mid obs) = sigmoid(\alpha_0 + n\alpha_1 + n_1\alpha_2 + n^2\alpha_5 + n_1 n\alpha_6 + n_1^2\alpha_7)$$

$r(X)$ is true for n_1 individuals out of a population of n.

Danger of fitting to data without understanding the model

- MLN/RLR can fit sigmoid of any polynomial.
- Consider sigmoid of polynomials of degree 2:

$$sigmoid(-0.01n^2 - 0.2n + 8)$$

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Both go from ≈ 1 at $n = 10$ to ≈ 0 at $n = 30$.
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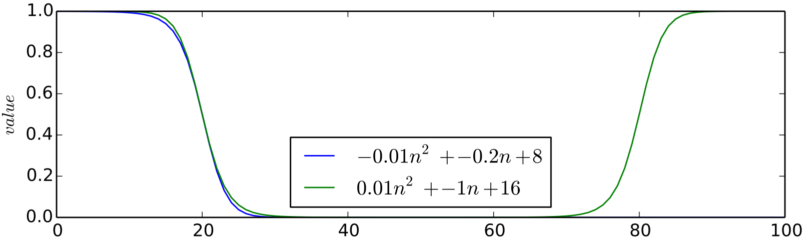

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There are unboundedly many possible relations in a real-world object such as a house.

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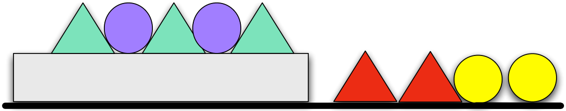
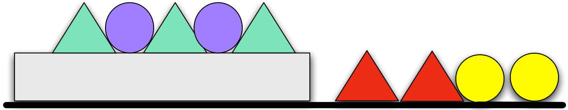


Observe a triangle and a circle touching. What is the probability the triangle is green?

$$P(green(x) \mid triangle(x) \land \exists y \; circle(y) \land touching(x, y))$$

The answer depends on how the x and y were chosen!

Protocol for Observing



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$select(x)$	$select(y)$	$select(x, y)$
$select(y)$	$select(x)$	
$3/4$	$2/3$	$4/5$

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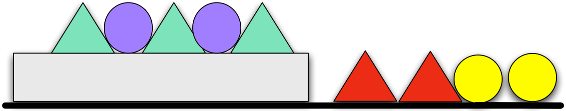

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A logical formula does not provide enough information to determine the probabilities.

Data

Real data is messy!

- Multiple levels of abstraction
- Multiple levels of detail
- Sometimes observations are abstract and lifted
  e.g., "3 people out of 300 in the audience asked a question".
- Uses the vocabulary from many ontologies
- Rich meta-data:
  - Who collected each datum? (identity and credentials)
  - Who transcribed the information?
  - What was the protocol used to collect the data? (Chosen at random or chosen because interesting?)
  - What were the controls — what was manipulated, when?
  - What sensors were used? What is their reliability and operating range?
  - What is the provenance of the data; what was done to it when?
- Errors, forgeries, . . .

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- Learn general knowledge as well as about particular individuals
- Use the meta-data of how data was collected
- Model protocol used to generate the observations
- Also model what is not observed (e.g., because it was redundant information, unimportant, false or unknown)

What is now required is to give the greatest possible development to mathematical logic, to allow to the full the importance of relations, and then to found upon this secure basis a new philosophical logic, which may hope to borrow some of the exactitude and certainty of its mathematical foundation. If this can be successfully accomplished, there is every reason to hope that the near future will be as great an epoch in pure philosophy as the immediate past has been in the principles of mathematics. Great triumphs inspire great hopes; and pure thought may achieve, within our generation, such results as will place our time, in this respect, on a level with the greatest age of Greece.

– Bertrand Russell 1917