Relational Probabilistic Models

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Work with: `http://minervaintelligence.com`, `https://treatment.com/`

March 20, 2019

Outline

1. Knowledge Graphs
   - Tensor Factorization and Neural Network Models
2. Representation Issues
   - Desiderata
   - How do relational models relate to probabilistic graphical models
3. Unique properties of relational models
   - Learning general knowledge vs learning about a data set
   - Varying Populations
   - What can be observed?
4. Conclusions and Challenges

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With a single relation it can be implicit $\longrightarrow$ triples:
$\langle pen_7, color, red\rangle$.

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- $prop(a, type, parcel)$, where *type* is a special property and *parcel* is a class.
- $prop(a, parcel, true)$, where *parcel* is a Boolean property (characteristic function of the class *parcel*).

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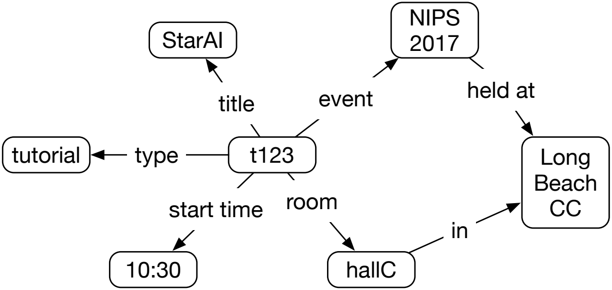
- Reify means: to make into an individual.
- How can we add extra arguments (e.g., presenters, chair)?

Triples and Knowledge Graphs

- Subject–verb–object
  Individual–property–value
  triples can be depicted as edges in graphs



- A modeller or learner needs to invent (reified) objects.

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All relations can be represented in terms of triples:

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	...	...	...
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$prop(Individual, Property, Value)$ is the only relation needed: $\langle Individual, Property, Value \rangle$ triples, Semantic network, entity relationship model, knowledge graphs, ...

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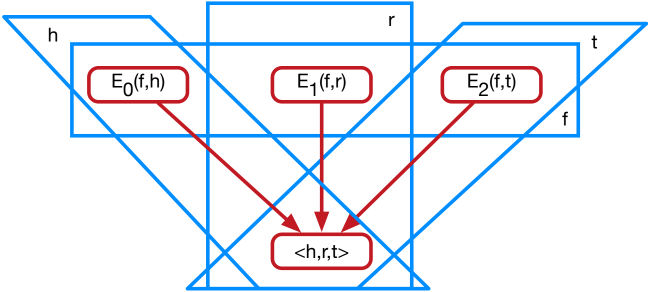

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- CompleX: the embeddings are complex numbers, tail is the conjugate of the head embedding
- SimpleE: have an embedding for r^{-1} and learn to predict both $\langle h, r, t\rangle$ and $\left\langle t, r^{-1}, h\right\rangle$

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- Which is better depends on the goals and how success is measured.
- Many of the methods try to do both; learn about specific individuals and general knowledge.

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- Often we compare rankings (ordering), but what if the answer is "no"?
- Difficult to learn about reified entities.

Predicting Properties

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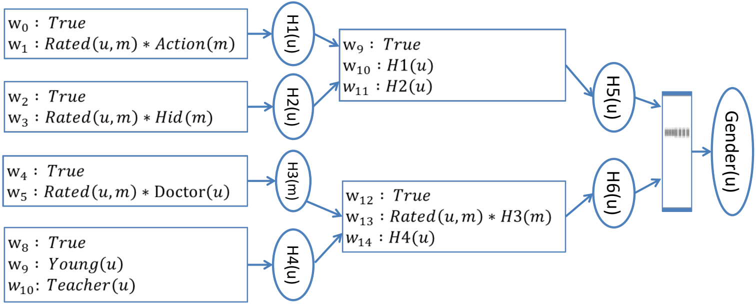
- We can build relational neural networks to solve this

[Kazemi & Poole, AAAI 2017]

Relational Neural Nets (RelNNs)



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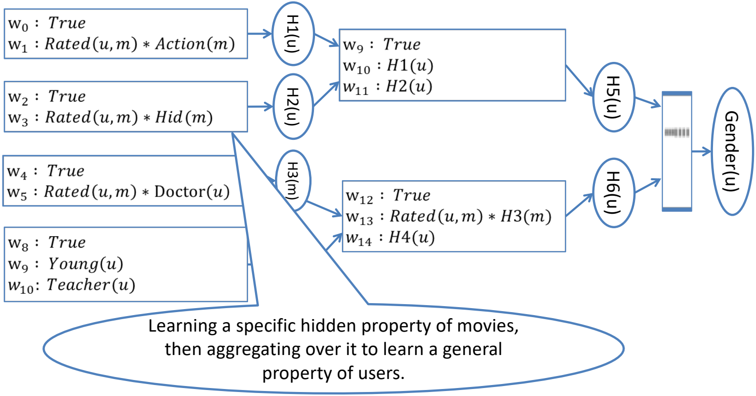

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- Modularity:
  Can independently developed parts be combined to form larger model?
  Can a larger model be decomposed into smaller parts?

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- Undirected models (Markov networks, factor graphs) represent probability distributions in terms of factors.
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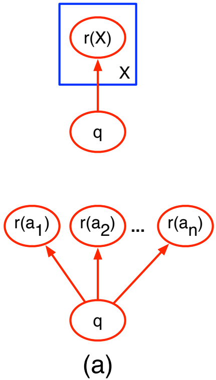
- That does not mean that representations for undirected models can represent directed models.

Three Elementary Models



(a)

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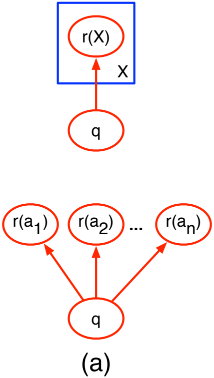

(a) Naïve Bayes
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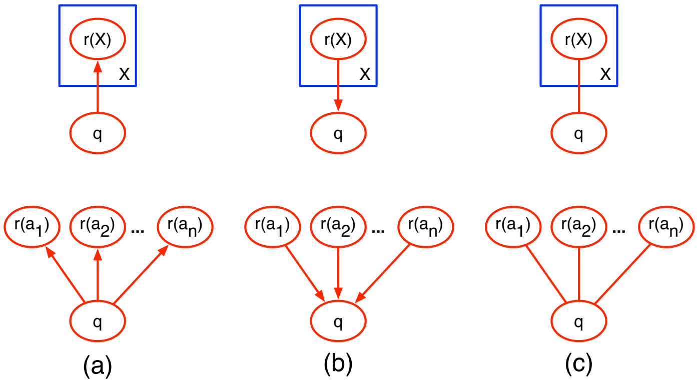

(a) Naïve Bayes
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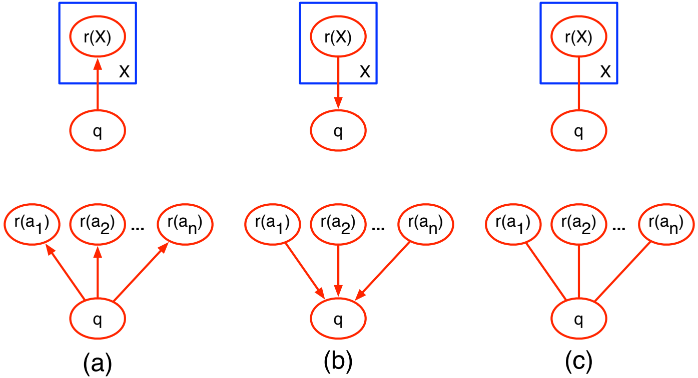

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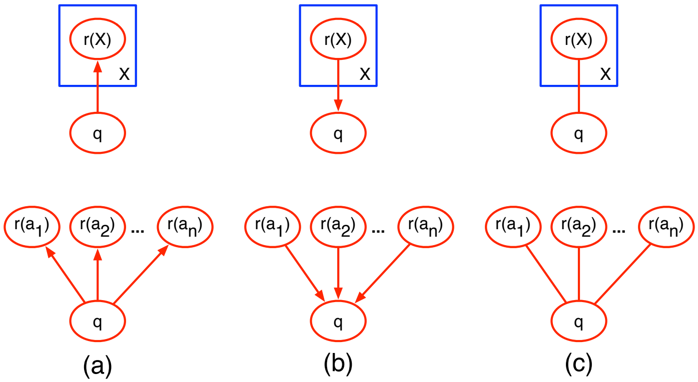

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— They are identical models when all *r*'s are observed.

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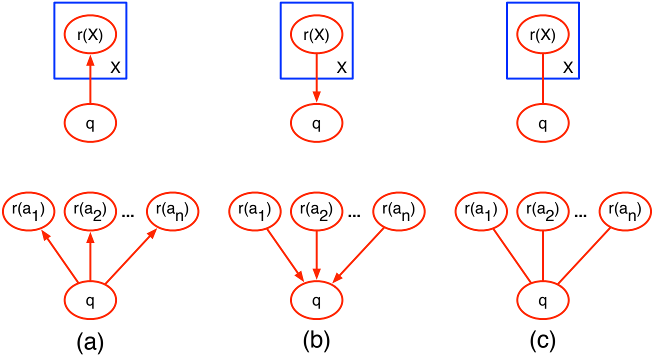

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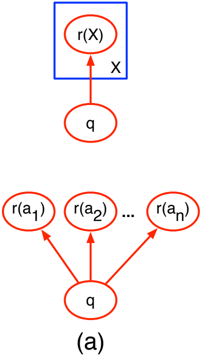

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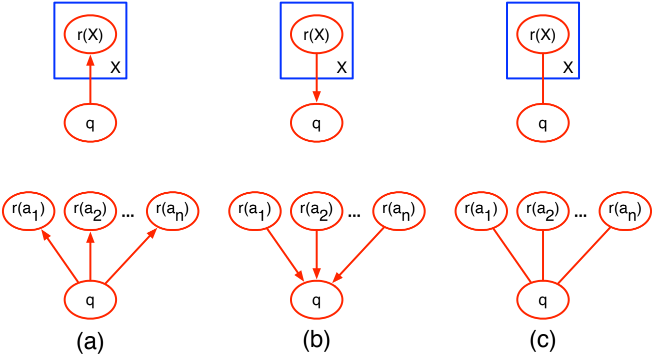

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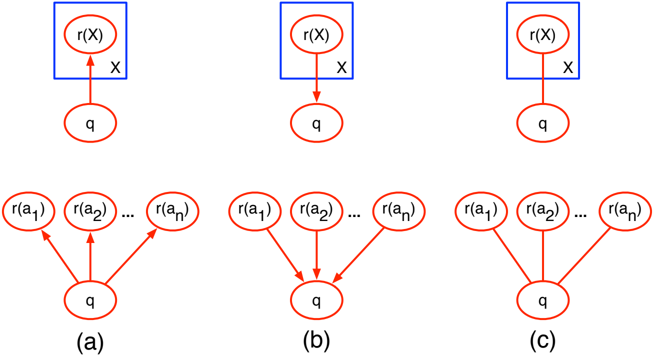

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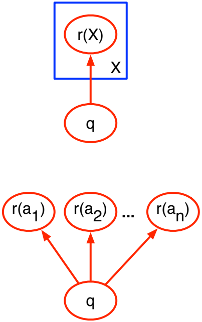

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  - if q is summed out, the distribution over $r(a_1) \ldots r(a_n)$ is not changed.
  - Why? requires factors on arbitrary subsets of $r(a_1) \ldots r(a_k)$
    — pairwise (or 3-wise or ...) interactions are not adequate
    — can't marry the parents

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  - Problog cannot represent negative effects: someone is less likely to smoke if their friends smoke (without there being a non-zero probability of logical inconsistency)

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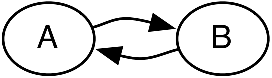
- Make model acyclic, by totally ordering variables. Destroys exchangeability. Symmetries are not preserved.
- (Relational) dependency networks: directed model,



  - $P(A, B)$ has 3 degrees of freedom,
  - $P(A \mid B), P(B \mid A)$, uses 4 numbers; typically inconsistent.
  - resulting distribution means stationary (equilibrium) distribution of Markov chain.

Outline

1. Knowledge Graphs
   - Tensor Factorization and Neural Network Models
2. Representation Issues
   - Desiderata
   - How do relational models relate to probabilistic graphical models
3. Unique properties of relational models
   - Learning general knowledge vs learning about a data set
   - Varying Populations
   - What can be observed?
4. Conclusions and Challenges

Learning general knowledge vs learning about a data set

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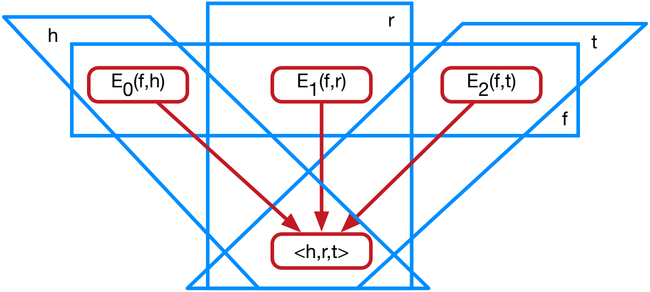
- Which is better depends on the goals and how success is measured.
- Many of the methods try to do both; learn about specific individuals and general knowledge.

Canonical polyadic tensor factorization model



$$P(\langle I, P, V\rangle = sigmoid(\sum_d h_s(I,d)h_v(P,d)h_o(V,d))$$

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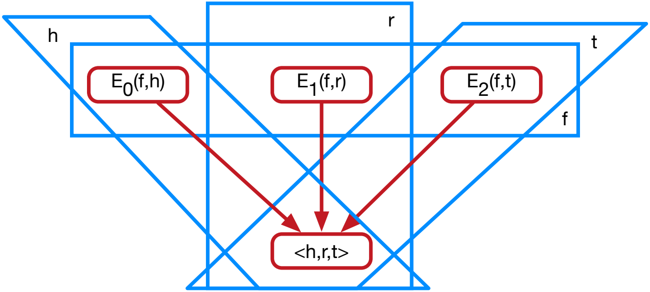


$$P(\langle I, P, V\rangle = sigmoid(\sum_d h_s(I,d)h_v(P,d)h_o(V,d))$$

- Doesn't learn generalized knowledge but about the particular individuals
- There are many more and less sophisticated models

Outline

What happens as Population size n Changes: Simplest case

$$\alpha_0 : q$$
$$\alpha_1 : q \wedge \neg r(x)$$
$$\alpha_2 : q \wedge r(x)$$
$$\alpha_3 : r(x)$$

Weighted formulae define distribution:

$$P_{MLN}(q \mid n) = sigmoid(\ \alpha_0 + n \log(e^{\alpha_2} + e^{\alpha_1 - \alpha_3})\ )$$

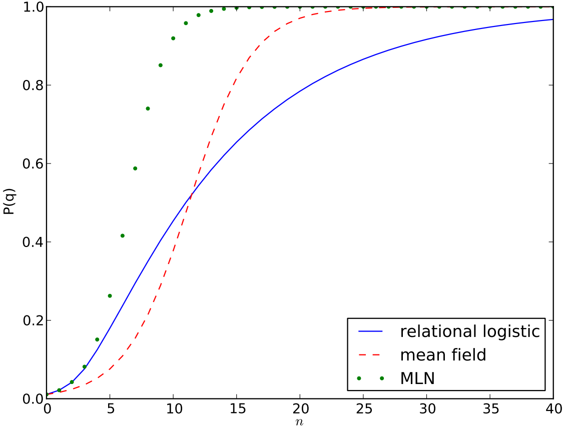
Weighted formulae define conditionals:

$$P_{RLR}(q \mid n) = \sum_{i=0}^{n} \binom{n}{i} sigmoid(\alpha_0 + i\alpha_1 + (n-i)\alpha_2)(1-p_r)^i p_r^{n-i}$$

Mean-field approximation:

$$P_{MF}(q \mid n) = sigmoid(\alpha_0 + np_r\alpha_1 + n(1-p_r)\alpha_2)$$

Population Growth: $P(q \mid n)$



All: $P(q \mid n) \rightarrow$ 0 or 1 as $n \rightarrow \infty$

Challenges of varying populations

- Example: The Movielens 100k dataset contains data about *rated*(P, M, R, T) meaning person P gave movie M a rating of R at time T.
  Plus user demographic and movie information.
- Number of ratings per user is between 20 (arbitrary threshold) and 737; average of 106
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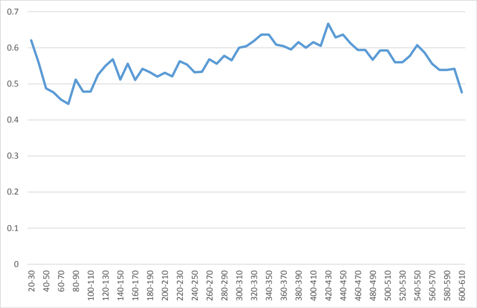
  - There is 100 times as much ratings information as age information
- Bigger datasets have even more variability.

Real Data



Observed $P(25 < Age(u) < 45 \mid n)$, where n is number of movies watched from the Movielens dataset.

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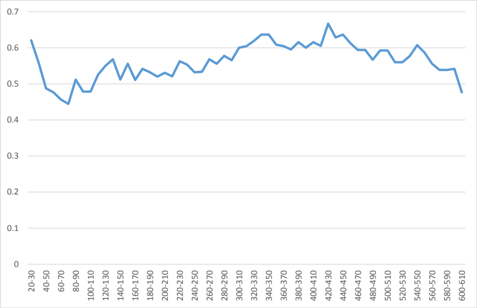


Observed $P(25 < Age(u) < 45 \mid n)$, where n is number of movies watched from the Movielens dataset.
Dont use:

$$w : middle\_age(U) \leftarrow rated(U, M) \wedge foo(M)$$

then $P(middle\_age(U)) \rightarrow$ 0 or 1 as number of movies increases.

Example of polynomial dependence of population

$\alpha_0 : q$
$\alpha_1 : q \wedge \mathit{true}(X)$
$\alpha_2 : q \wedge r(X)$
$\alpha_3 : \mathit{true}(X)$
$\alpha_4 : r(X)$
$\alpha_5 : q \wedge \mathit{true}(X) \wedge \mathit{true}(Y)$
$\alpha_6 : q \wedge r(X) \wedge \mathit{true}(Y)$
$\alpha_7 : q \wedge r(X) \wedge r(Y)$

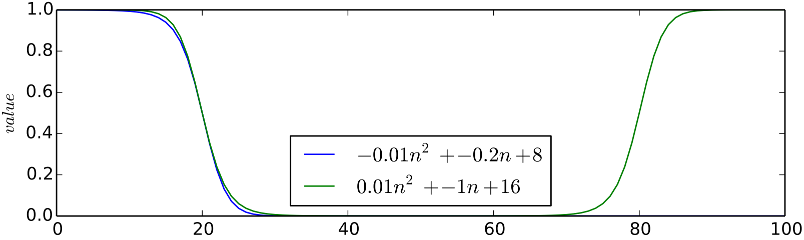
In RLR and in MLN, if all $r(a_i)$ are observed:

$$P(q \mid obs) = \mathit{sigmoid}(\alpha_0 + n\alpha_1 + n_1\alpha_2 + n^2\alpha_5 + n_1 n\alpha_6 + n_1^2\alpha_7)$$

$r(X)$ is true for n_1 individuals out of a population of n.

Danger of fitting to data without understanding the model

- RLR can fit sigmoid of any polynomial.
- Consider a polynomial of degree 2:



Outline

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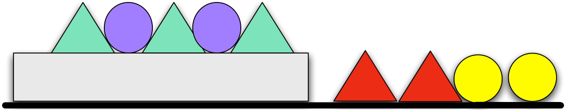
There are unboundedly many possible relations in a real-world object such as a house.

Observation Protocols

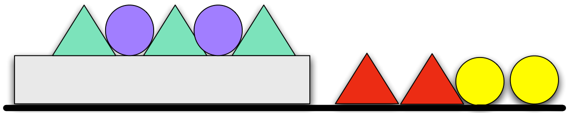


Observe a triangle and a circle touching. What is the probability the triangle is green?

$$P(green(x) \mid triangle(x) \land \exists y \; circle(y) \land touching(x, y))$$

The answer depends on how the x and y were chosen!

Protocol for Observing



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$select(x)$	$select(y)$	$select(x, y)$
$select(y)$	$select(x)$	
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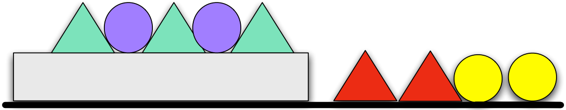

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A logical formula does not provide enough information to determine the probabilities.

Data

Real data is messy!

- Multiple levels of abstraction
- Multiple levels of detail
- Sometimes observations are abstract and lifted
  e.g., "3 people out of 300 in the audience asked a question".
- Uses the vocabulary from many ontologies
- Rich meta-data:
  - Who collected each datum? (identity and credentials)
  - Who transcribed the information?
  - What was the protocol used to collect the data? (Chosen at random or chosen because interesting?)
  - What were the controls — what was manipulated, when?
  - What sensors were used? What is their reliability and operating range?
  - What is the provenance of the data; what was done to it when?
- Errors, forgeries, . . .

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- Learn general knowledge as well as about particular individuals
- Use the meta-data of how data was collected
- Model protocol used to generate the observations
- Also model what is not observed (e.g., because it was redundant information, unimportant, false or unknown)