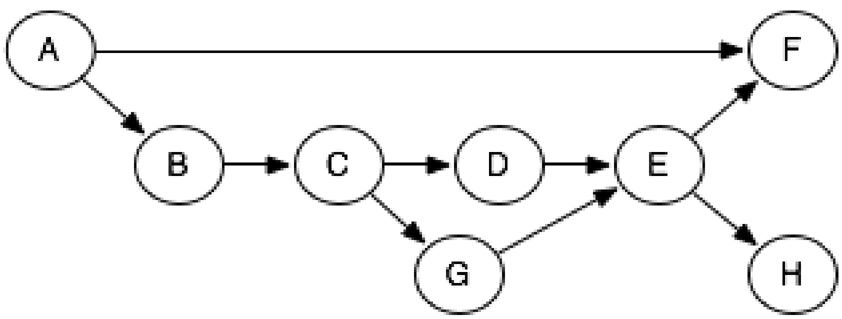
CPSC 522 — Spring 2012

Assignment 3 — solution

Question 1

Consider the following belief net:



(a) Consider the query: $P(G|F = true, H = false)$. For the elimination ordering, B, D, E, A, C give all of factors created in VE. For each factor created, specify which factors were removed, and what variable was summed out. You do not need to consider any numerical values.

Solution:

Var Eliminated	Factors Removed	Factor Created
B	$P(C\|B)P(B\|A)$	$f_1(A,C)$
D	$P(D\|C)P(E\|D,G)$	$f_2(C,E,G)$
E	$P(f\|E,A)P(\neg h\|E))f_2(C,E,G)$	$f_3(A,C,G)$
A	$P(A)f_1(A,C)f_3(A,C,G)$	$f_4(C,G)$
C	$P(G\|C)f_4(C,G)$	$f_5(G)$

(b) Consider using recursive conditioning for the same query, assuming that the variable were split in the order G, C, A, E, D, B. What values are used from the cache in this computation? Do any of the assignments disconnect the graph? If so, which ones?

Solution: From the previous part we see that the factorization is:

$$P(G,f,\neg h) = \sum_C P(G|C) \sum_A P(A) \left(\sum_B P(C|B)P(B|A) \right) \left(\sum_E (f|EA)P(\neg h|E) \sum_D P(D|C)P(E|D,G) \right)$$

The cached values stored after summing out D for one value of A can be used for the other value of A, as the factors do not depend on A. However they will not be used for other values of C, E or G as the values in the factors depend on these variables.

The cached value stored after summing out B can be used for the other value of G, but not for other values of C or A.

After assigning G, C and A the factors containing $\{P(C = v_1|B), P(B|A = v_2)\}$ are disconnected from the other factors.

Question 2

Suppose we have a relational probabilistic model for movie prediction, where we represent

$P(likes(P,M)|age(P), genre(M))$

(a) What is the treewidth of the ground belief network (after pruning irrelevant variables) for querying $age(Sam)$ given the following observations?

Person	Movies	likes
Sam	Hugo	yes
Chris	Hugo	no
Sam	The Help	no
Sam	Harry Potter 6	yes
Chris	Harry Potter 6	yes
Chris	AI	no
David	AI	yes
David	The Help	yes

(b) For the same probabilistic model, for m movies, n people and r ratings, what is the worst-case treewidth of the corresponding graph (after pruning irrelevant variables), where only ratings are observed? (Here is it worst over the set of all observations).

(c) For the same probabilistic model, for m movies, n people, and r ratings, what is the worst-case treewidth of the corresponding graph, where the some ratings but all of the genres are observed?

Solution:

(a) 2, e.g., by eliminating in order $genre(AI)$, $age(David)$, $genre(HP6)$, $genre(TheHelp)$, $genre(Hugo)$, $age(Chris)$

(b) $min(\lfloor\sqrt{r}\rfloor, m, n)$, which occurs when the r ratings are used to connect each of $\lfloor\sqrt{r}\rfloor$ movies with each of $\lfloor\sqrt{r}\rfloor$ people.

(c) 0, as the age nodes are disconnected.

Question 3

For this question you should use AILog (see `http://artint.info/code/ailog/ailog_man.html`).

Consider the electrical domain of Figure 5.2 of the textbook. Using the relations of Example 12.11 of the textbook and the probabilities of the AIspace "electrical diagnosis problem", write an AILog program that computes the same posterior probabilities as the belief network. Make the rules as general as possible, so that your axiomatization can be applied to different configurations. You need to hand in a documented program and evidence that it works.

Solution: A solution is at `http://artint.info/code/ailog/ailog_code/ch14/elect_relational_prob.ailog`. This gives the same answers as the AIspace code at `http://artint.info/code/ailog/ailog_code/ch14/elect_aispace.xml`. Note that this is slightly modified from the original aispace code to make it consistent (so the switches all work the same).

Question 4

How long did the assignment take? What did you learn? Was it reasonable? What suggestions do you have to improve the assignment?

It should have taken a few (around 10) hours. You were supposed to learn about exact inference, and about representing and reasoning with relational probabilistic models.