## CPSC 502 - Fall 2013 <br> Assignment 3

Due: 10:00pm, Sunday 20 October 2013.
This can be done in groups of size 1,2 or 3 . Working alone is not recommended. A group of size $n$ can choose any $n+1$ questions from questions 1-6. All members of the group need to be able to explain the group's answer. Please look at all of the questions, as the exam will assume that you have thought about all of the questions. Everyone should do question 7 (it is worth marks). Please post questions to the Connect web site.

## Question 1

Consider the domain of house plumbing represented in the diagram of Figure 1. This is the same domain used in the previous assignment.

In this figure, $p 1, p 2$ and $p 3$ denote cold water pipes. $p 1$ is the pipe coming in from the main water supply. $t 1, t 2$ and $t 3$ are taps and $d 1, d 2$ and $d 3$ are drainage pipes. The constants shower denotes a shower, bath denotes a bath, sink denotes a sink, and floor denotes the floor. There can also be plugs in the sink or in the bath. You can assume that you are in a static situation (i.e., you don't have to worry about time); imagine that you have stumbled in this situation and must reason about it.

Suppose you can observe or query the tap positions, flow out of drain d1, and whether there is water in the sink and bath, and whether there is water on the floor.


Figure 1: The Plumbing Domain
(a) What are the random variables? In particular, you need random variables to represent the observations you may want to make, the queries you may be interested and other (hidden) variables to keep the model simple. For each variable, give its domain and intended interpretation.
(b) Give a belief network for these variables, assuming a causal ordering of the variables. Give reasonable conditional probability tables. You can use the AISpace applet or any other belief network tool.
(c) Suppose you had to demonstrate to a skeptic that belief networks are appropriate for this domain. Argue that this is a reasonable representation, and as part of your explanation, you need to give some test cases that show what your model is capable of. Use proper English. Be brief and concise (as this skeptic doesn't have much time).
(d) What would your belief network look like if you had chosen the opposite ordering of variables? (Explain, with examples.)

## Question 2

Consider the belief network:

with Boolean variables (we will write $A=$ true as $a$ and $A=$ false as $\neg a$ ) and the following conditional probabilities:

$$
\begin{array}{rlrl}
P(a) & =0.1 & P(d \mid b) & =0.1 \\
P(b) & =0.8 & P(d \mid \neg b) & =0.8 \\
P(c \mid a, b) & =0.9 & P(e \mid c) & =0.7 \\
P(c \mid a, \neg b) & =0.8 & P(e \mid \neg c) & =0.2 \\
P(c \mid \neg a, b) & =0.3 & P(f \mid c) & =0.2 \\
P(c \mid \neg a, \neg b) & =0.4 & P(f \mid \neg c) & =0.9
\end{array}
$$

(a) Compute $P(e)$ using variable elimination. Please try to do it by hand, then check it with the AISpace applet. Do not do any pruning. Explain the significance of each factor created (either what does it represent or why does it have particular values). Show only what factors are created; don't show the tables.
(b) Compute $P(e \mid \neg f)$ using variable elimination. How much of the previous computation can be reused? Show only what is different.
(c) Compute $P(a \mid d)$ using variable elimination.
(d) Compute $P(a \mid \neg f)$.
(e) Compute $P(a \mid \neg f, d)$.
(f) Explain when observing $d$ affects your belief in $a$. (Be as specific as possible.)

## Question 3

Assume you are part of a team writing a open textbook for students learning AI. Your job is to write examples that explain the concepts below. You are to show how to compute $P(f \mid d)$ in the belief network of the previous question. Your explanation should highlight why each method may be good for this example. (If there is a choice in how the algorithm works make the choice that showcases the method.)
(a) Rejection sampling
(b) Importance sampling
(c) Particle filtering

## Question 4

D. Kahneman [Thinking Fast and Slow, 2011, p. 166] gives the following example:
"A cab was involved in a hit-and-run accident at night. Two cab companies, the Green and the Blue, operate in the city. You are given the following data:

- $85 \%$ of the cabs in the city are Green and $15 \%$ are Blue.
- A witness identified the cab as Blue. The court tested the reliability of the witness in the circumstances that existed on the night of the accident and concluded that the witness correctly identifies each one of the two colours $80 \%$ of the time and failed $20 \%$ of the time.

What is the probability that the cab involved in the accident was Blue?"
(a) Represent this story as a belief network. Explain all variables and conditional probabilities. What is observed, what is the answer?
(b) Suppose there were three independent witnesses, two of which claimed the cab was Blue and one of whom claimed the cab was Green. Show the corresponding belief network. What is the probability the cab was Blue? (What if all three claimed the cab was Blue?)
(c) Suppose it was found that the two witnesses who claimed the cab was Blue were not independent, but there was a $60 \%$ chance they colluded. (What might this mean?) Show the corresponding belief network, and the relevant probabilities. What is the probability that the cab is Blue, (both for the case where all three witnesses claim that cab was Blue and the case where the other witness claimed the cab was Green)?
(d) In a variant of this scenario, Kahneman [p 167] replaced the first condition with: "The two companies operate the same number of cabs, but Green cabs are involved in $85 \%$ of the accidents." How can this new scenario be represented as a belief network? Show how it works. Be explicit about any assumptions you make.

## Question 5

Give a possible exam question (perhaps with sub-parts) that would be good to test students about representing or reasoning with belief networks. It should be worth 10 marks, and take students approximately 10 minutes to complete in an exam setting. It must be clear what the question is asking for and must be self-contained. Give a solution.

## Question 6

On the wiki http://wiki.ubc.ca/Course:CPSC:Artificial_Intelligence create pedagogical or real-world examples that use useful for students to learn about probabilistic reasoning. Please add references for real-world examples. You will need to login with your CWL to edit. This is intended to be an open-ended creative question. This is a cooperative question, as anyone can edit other people's questions. It is possible to gain credit by improving other's contributions. Please help to build a useful resource.

It can be worth multiple questions in this part; please justify any claim of how many questions your contribution is worth. It is even possible to do the whole assignment just by creating useful resources.

## Question 7

For each question in this assignment, say how long you spent on it. Was this reasonable? What did you learn?

