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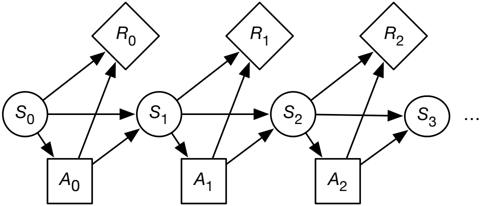
- Variable elimination computes the posterior probability of a variable given evidence by summing out the non-observed non-query variables.
- A causal model predicts the effect of a intervention.
- A naive bayes model can be used for learning and help systems.
- Utility is a measure of preferences that is defined in terms of lotteries.
- A decision network has random nodes, decision nodes and a utility node. A policy has an expected utility.
- A Markov decision process can model ongoing activity with rewards, but only fully observable case is feasible.

Markov Decision Processes



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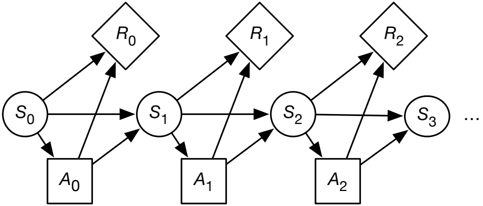
Markov Decision Processes



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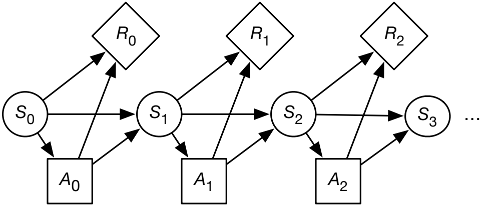

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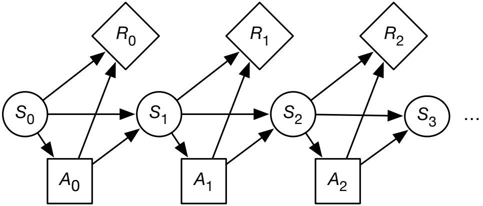

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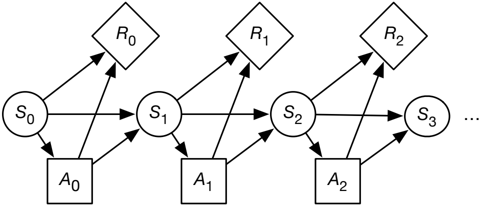

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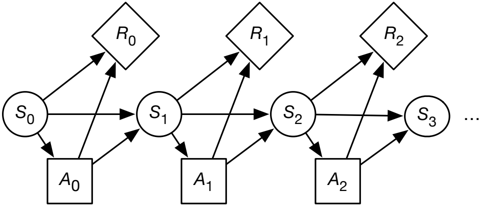

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Example: Go (the game)

- Go is a board game on 19×19 grid. Each position can have black stone or white stone or no stone.
- Approximately $2 * 10^{170}$ states (legal board positions)

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- Use neural networks to represent $V(s)$ and $\pi(s)$ (see CPSC 340)

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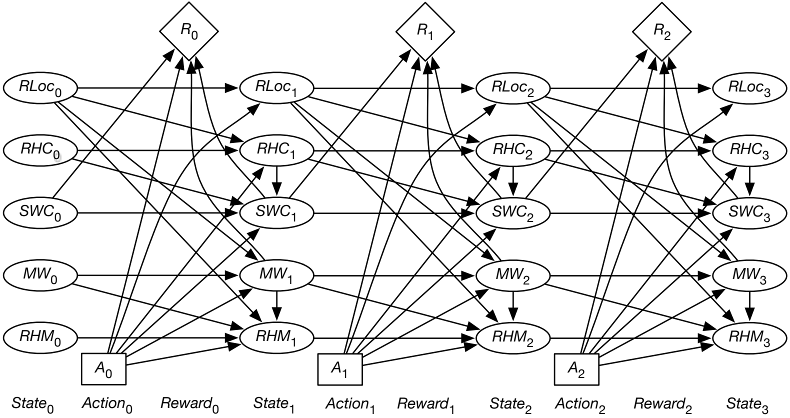
- Two players that are adversaries. (Multiple-agent reasoning is *much* more difficult with partial observability (eg. simple interaction such as poker).

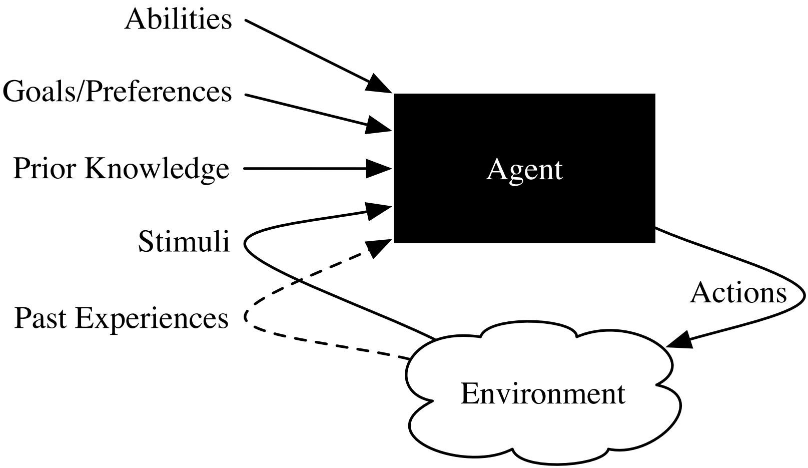
Dynamic Decision Network



Parents of A_i are all of the $State_i$ variables.

Overview

Agents acting in an environment



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Classical Forward/Regression Planning

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- There are many variants of local search to find solutions or minimize an evaluation function

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- A CSP planner converts a planning problem into a CSP for a fixed planning horizon.

Probability and Belief Networks

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Decision Networks

Decision network:

- DAG with three sorts of nodes: decision (rectangle), random (ellipse), utility (diamond)
- Domain for each decision and random node (no domain for utility)
- Factor for each random node and the utility (no initial factor for decision nodes)

Decision Networks

Decision network:

- DAG with three sorts of nodes: decision (rectangle), random (ellipse), utility (diamond)
- Domain for each decision and random node (no domain for utility)
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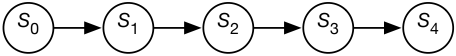
- Find optimal policy: sum out random variables until a decision variable D is in a factor F where all of the other variables in F are parents of D; then maximize D.

Markov Models

- A Markov chain is a particular sort of belief network that can extend indefinitely:



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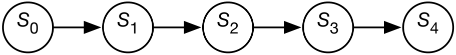

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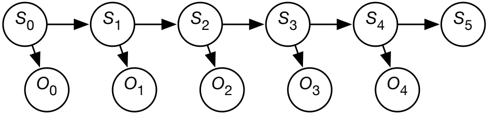

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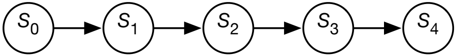

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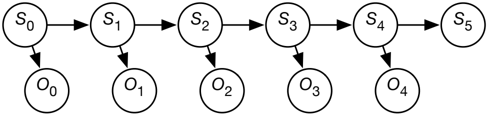
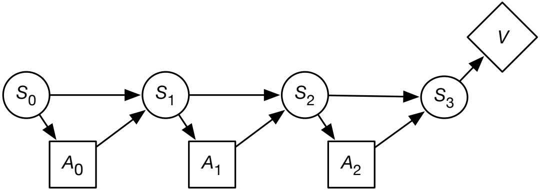


- Hidden Markov models can be used for localization and simple language models

Markov Decision Process (MDP)

An MDP combines decision network and Markov chain:



- Initial Factors:

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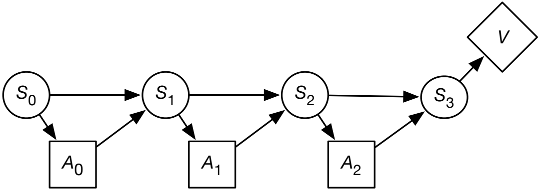

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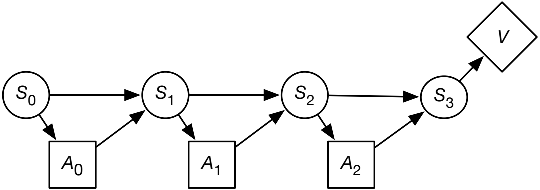

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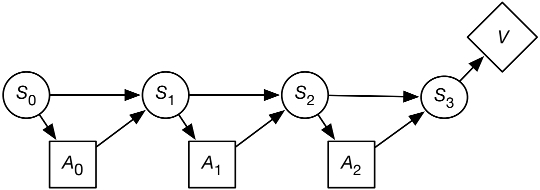

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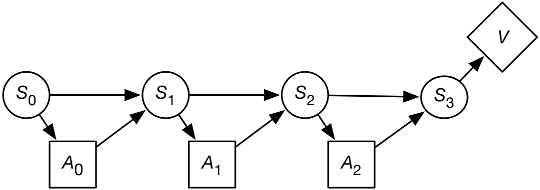

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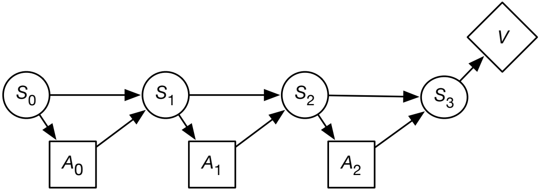

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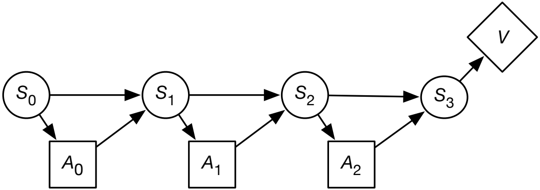

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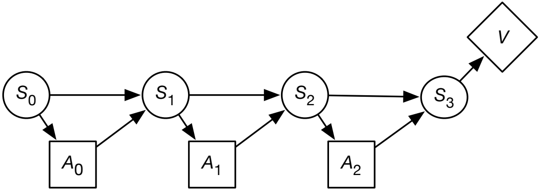

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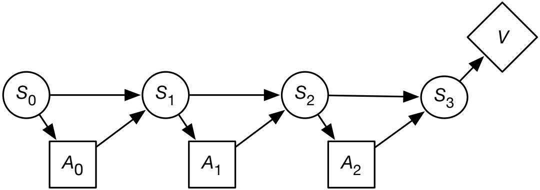

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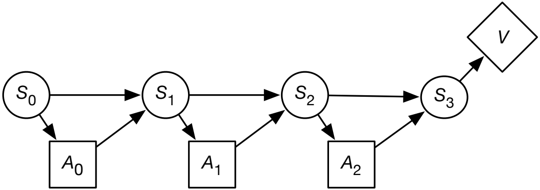

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- This is the same form as the original problem, with one less time step $\rightarrow$ solve recursively.
- This works for arbitrary stages (number of times)
- An MDP extends indefinitely, and often includes rewards at each time. Reinforcement learning typically works by estimating $Q(S, A)$. Assumes fully observable environment.

Overview of Course

	dynamics	observable	repr	stage	preference	rationality
search	det	fully	states	indef	goals	perfect
CSPs	det	fully	feats	static	—	perfect
SLS	det	fully	feats	static	—	bounded
planning	det	fully	feats	indef	goals	perfect
belief nets	stoch	partial	feats	static	—	perfect
stoch siml	stoch	partial	feats	static	—	bounded
decision nets	stoch	partial	feats	finite	utility	perfect
Markov models	stoch	partial	states	infinite	—	perfect
MDP	stoch	fully	states	infinite	utility	perfect