Announcements

- Solution to Assignment 3 is posted
- Assignment 4 is available. Use AISpace 1 or AIPython; AISpace 2 is a graphical tracer for AIPython and is not necessary.
- Midterm next Thursday.
  - 75 minutes anytime in 24 hour period.
  - Invidualized exams.
  - You may use programs and the Intenet, but you may not not consult or talk to anyone about the exam.
  - Be prepared for an oral exam after to explain how you got your answer.

Review: So far...

- Constraint satisfaction problems are defined in terms of variables, domains, constraints
- Constraint satisfactions problems can be solved with:
  - Search
  - Arc consistency with domain splitting
  - Local search
- Local search maintains a complete assignment of a value to each variable, and has a mix of improving and randomized steps.

Today: Learning Objectives

Today: Local Search

At the end of the class you should be able to:

- show how a CSP can be solved using local search
- compare stochastic algorithms
- explain how randomness helps
- know a bit about population methods

Local Search

Local Search:

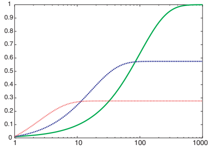
- Maintain a complete assignment of a value to each variable.
- Start with random assignment (or a good guess)
- Repeat:
  - Select a variable to change
  - Select a new value for that variable
- Until a satisfying assignment is found

Runtime Distribution

- Run the same algorithm on the same instance for a number of trials (e.g., 100 or 1000)
- Sort the trials according to the run time.
- Plot:

  x-axis run time of the trial
  y-axis index of the trial

  This produces a cumulative distribution



- Do this this a few times to gauge the variability (take a statistics course!)
- Sometimes use number of steps instead of run time (because computers measure small run times inaccurately) . . . not good measure to compare algorithms if steps take different times

Greedy Descent Variants

To select a variable to change and a new value for it:

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  What needs to be done at every step?
- Select a variable and value at random; accept this change if it doesn't increase the number of conflicts.

Clicker Question

Which of the following is true:

A If an algorithm is above and to the left of another algorithm in a runtime distribution, it is always faster

B A random walk cannot escape a local minima

C The amount of time taken per step is about the same for all local search methods given modern data structures and the speed of computers

D Carrying out arc consistency before doing a local search can reduce the search space

Variant: Simulated Annealing

- Pick a variable at random and a new value at random.
- If it isn't worse, accept it.
- If it is worse, accept it probabilistically depending on a temperature parameter, T:
  - With current assignment A and proposed assignment A' accept A' with probability $e^{(h(A)-h(A'))/T}$

  Note: $h(A) - h(A')$ is negative if A' is worse
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Probability of accepting a change:

Temperature	1-worse	2-worse	3-worse
10	0.91	0.81	0.74
1	0.37	0.14	0.05
0.25	0.02	0.0003	0.000006
0.1	0.00005	2×10^{-9}	9×10^{-14}

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n	$p=0.1$	$p=0.3$	$p=0.5$	$p=0.8$
5	0.410	0.832	0.969	0.9997
10	0.65	0.971	0.9990	0.9999998
20	0.878	0.9992	0.9999991	0.99999999999
50	0.995	0.99999998	0.9999999999999991	1.0

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- It can be expensive if k is large.

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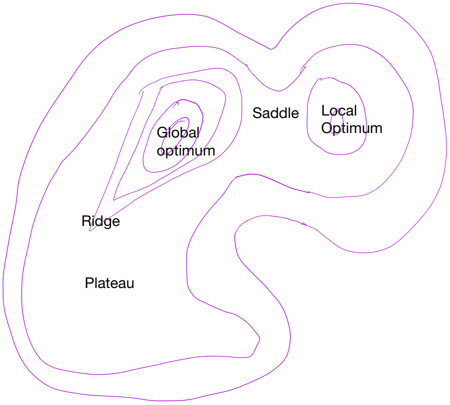
  η is the step size.
- Neural networks do gradient descent with thousands or millions or billions of dimensions to minimize error on a dataset. (See CPSC 340).

Problems with Greedy Descent

- a local optimum that is not a global optimum



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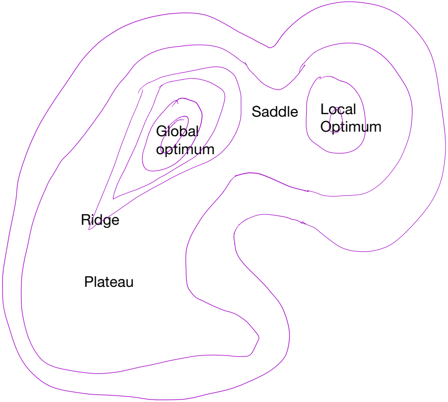

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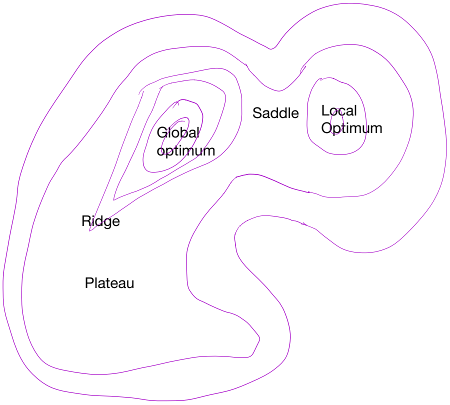

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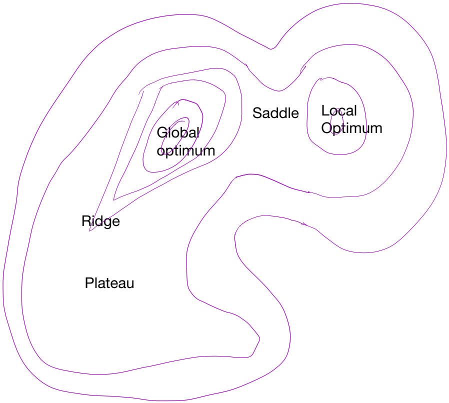
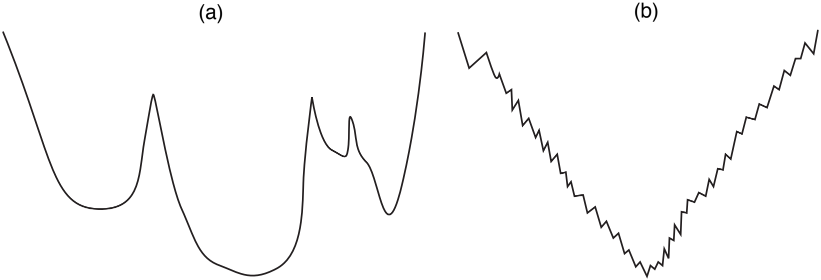
- a saddle is a flat area where steps need to change direction



1-Dimensional Ordered Examples

Two 1-dimensional search spaces; small step right or left:



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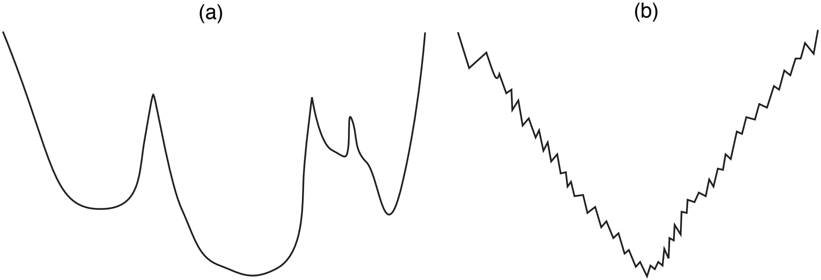

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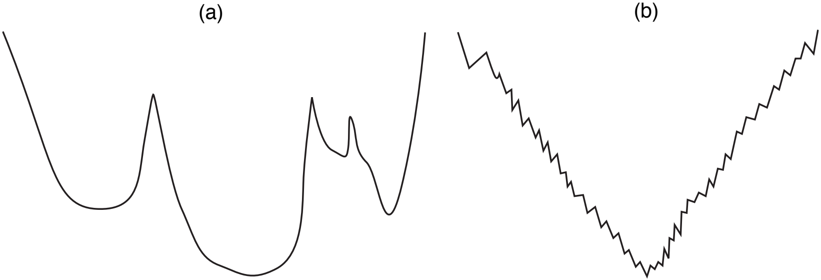

- Which method would most easily find the global minimum?
- What happens in hundreds or thousands of dimensions?
- What if different parts of the search space have different structure?

Parallel Search

A total assignment is called an individual.

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- Like k restarts, but uses k times the *minimum* number of steps.

Beam Search

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- Problem: lack of diversity of individuals.

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- Like asexual reproduction: each individual mutates and the fittest ones survive.

Genetic Algorithms

- Like stochastic beam search, but pairs of individuals are combined to create the offspring.
- For each generation:
  - Randomly choose pairs of individuals where the fittest individuals are more likely to be chosen.
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  - Mutate some values.
- Stop when a solution is found.

Crossover

- Given two individuals:

$$X_1 = a_1, X_2 = a_2, \ldots, X_m = a_m$$

$$X_1 = b_1, X_2 = b_2, \ldots, X_m = b_m$$

- Select i at random.
- Form two offspring:

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- The effectiveness depends on the ordering of the variables.
- Many variations are possible.

Clicker Question

Which of the following is false:

- A Population based methods carry out multiple local searches at once
- B The time taken by population-based methods is number of individuals (local searches) multiplied by the *minimum* time one the local searches finds a solution
- C Crossover with selecting fittest individuals allows genetic algorithms to combine good parts of potential solutions
- D It is more likely that a population-based method will find a solution than a local search with no restart
- E Population-based methods are guaranteed to find a solution if there is one, even without randomness

Optimization

An optimization problem is given

- a set of variables, each with an associated domain
- an objective function that maps total assignments to real numbers, and
- an optimality criterion, which is typically to find a total assignment that minimizes (or maximizes) the objective function.

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- Arc consistency can be used to prune dominated values
- Can use local search
- Problem: we can't tell if a value is a global minimum unless we do systematic search

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  A literal is an atom or the negation of an atom.
- Clausal constraints: a clause is an expression of the form $l_1 \vee l_2 \vee \cdots \vee l_k$, where each l_i is a literal, and $\vee$ means "or".
  The clause is true if at least one of the l_i is true.

Representing finite CSPs as SAT problems

It is possible to convert any finite CSP into a propositional satisfiable problem:

- A variable Y with domain $\{v_1, \ldots, v_k\}$ can be converted into k Boolean variables $\{Y_1, \ldots, Y_k\}$, where Y_i is true when Y has value v_i and is false otherwise.
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  Often we can much more concise.

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- If we get to a clause with one element, we can assign the corresponding Boolean variable.
- If all of the literals in a clause are removed, there is no solution.

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Arc consistency can be made much more efficient in SAT problems than for general CSPs.

- Because domains are binary, pruning a domain is equivalent to assigning a value to the variable.
- If X is assigned *true*, all of the clauses with x can be removed, as they are all satisfied.
- If X is assigned *true*, all of the clauses with $\neg x$, of the form $\neg x \vee w$ can be simplified to w.
- If we get to a clause with one element, we can assign the corresponding Boolean variable.
- If all of the literals in a clause are removed, there is no solution.

— uniformity of the constraints means efficient data structures.

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  This allows for efficient indexing of clauses.
- The search space is expanded. Before a solution has been found, more than one of the indicator variables for a variable Y could be true, or all of the indicator variables could be false.