Announcements

- Solution to Assignment 3 is posted
- Assignment 4 is available
- Midterm next Thursday.
  - 75 minutes anytime in 24 hour period.
  - Invidualized exams.
  - You may use programs and the Intenet, but you may not not consult or talk to anyone about the exam.
  - Be prepared for an oral exam after to explain how you got your answer.

Review: So far...

- Constraint satisfaction problems are defined in terms of variables, domains, constraints
- Constraint satisfactions problems can be solved with:
  - Search
  - Arc consistency with domain splitting
  - Local search (today!)
- Local search maintains a complete assignment of a value to each variable, and has a mix of improving and randomized steps.

Today: Learning Objectives

Today:

- Assignment 3 solution discussion
- Local Search

At the end of the class you should be able to:

- show how a CSP can be solved using local search
- compare stochastic algorithms
- explain how randomness works

Assignment 3 solution

Clicker Question

Which of the following is false:

A If there is a solution, arc consistency will not make any domains empty

B Arc consistency always halts for finite CSPs

C Arc consistency involves checking constraints multiple times

D Arc consistency always results in singleton domains if there is only one solution.

Clicker Question

What is not true of arc consistency with domain splitting:

A Arc consistency needs domain splitting to solve problems in general

B Together they can solve CSPs in polynomial time

C They typically result in a smaller search space than using search without arc consistency

D They always terminate with a solution if there is one for finite CSPs

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  - Select a variable to change
  - Select a new value for that variable
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- The goal is an assignment with zero conflicts.
- Function to be minimized: the number of conflicts.

Iterative Best Improvement (2 stage) "greedy descent"

- Start with random assignment (for each variable, select a value for that variable at random)
- Repeat:
  - Select a variable that participates in the most conflicts
  - Select a different value for that variable
- Until a satisfying assignment is found

All selections are random and uniform.

Any Conflict

- Start with random assignment (for each variable, select a value for that variable at random)
- Repeat:
  - Select a variable at random that participates in any conflict
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- How can you compare three algorithms when
  - one solves the problem 30% of the time very quickly but doesn't halt for the other 70% of the cases
  - one solves 60% of the cases reasonably quickly but doesn't solve the rest
  - one solves the problem in 100% of the cases, but slowly?

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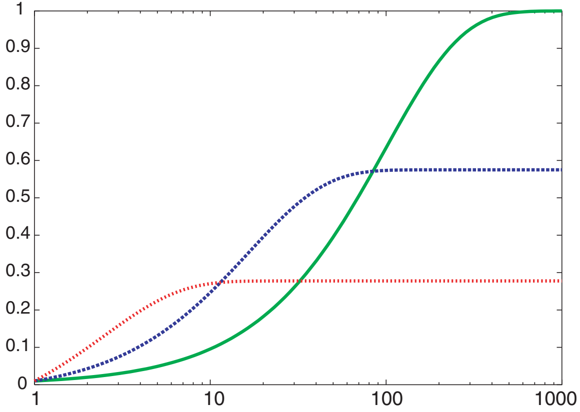
  - one solves the problem in 100% of the cases, but slowly?
- Summary statistics, such as mean run time, median run time, and mode run time don't make much sense.

Runtime Distribution

x-axis runtime (or number of steps)

y-axis the proportion (or number) of runs that are solved within that runtime



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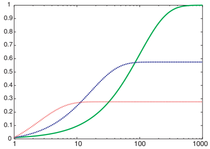
- Sometimes use number of steps instead of run time (because computers measure small run times inaccurately) . . . not good measure to compare algorithms if steps take different times



Randomized Algorithms

- A probabilistic mix of *greedy* and *any-conflict* — e.g., 70% of time pick best variable, otherwise pick any variable in a conflict – works better than either alone.

Stochastic Local Search

Stochastic local search is a mix of:

- Greedy descent: pick the best variable and/or value
- Random walk: picking variables and values at random
- Random restart: reassigning values to all variables

Some of these might be more complex than the others.
A probabilistic mix might work better.

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- If A is a total assignment, define $h(A)$ to be a measure of the difficulty of solving problem from A.
- $h(A) = 0$ then A a solution; lower h is better

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- Temperature can be reduced.

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n	$p=0.1$	$p=0.3$	$p=0.5$	$p=0.8$
5	0.410	0.832	0.969	0.9997
10	0.65	0.971	0.9990	0.9999998
20	0.878	0.9992	0.9999991	0.99999999999
50	0.995	0.99999998	0.9999999999999991	1.0

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- It can be expensive if k is large.

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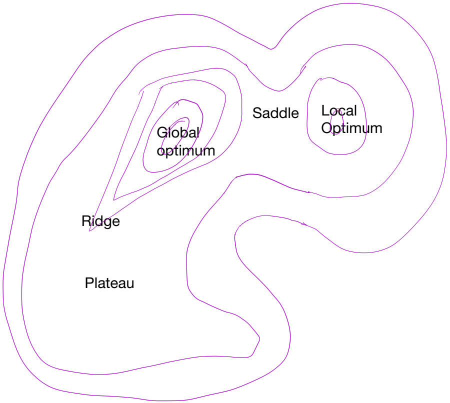
  η is the step size.
- Neural networks do gradient descent with thousands or millions or billions of dimensions to minimize error on a dataset. (See CPSC 340).

Problems with Greedy Descent

- a local optimum that is not a global optimum



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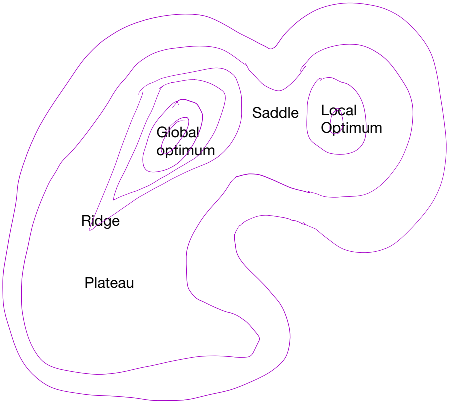

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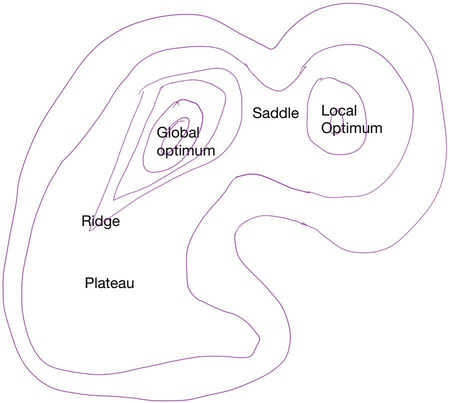

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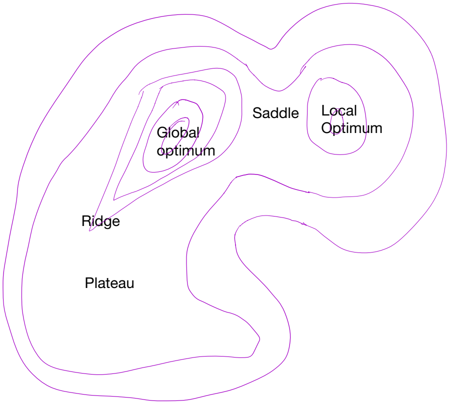
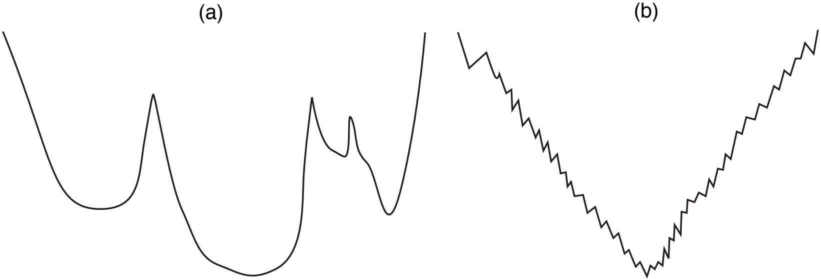
- a saddle is a flat area where steps need to change direction



1-Dimensional Ordered Examples

Two 1-dimensional search spaces; small step right or left:



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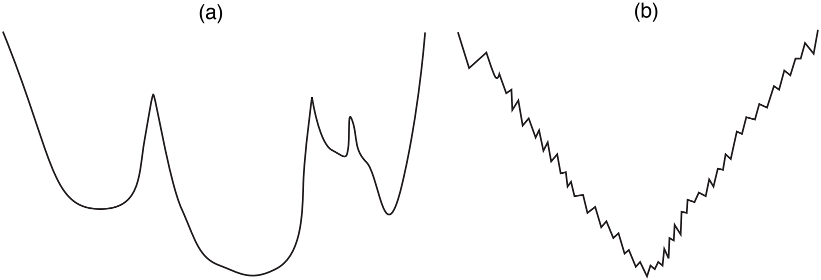

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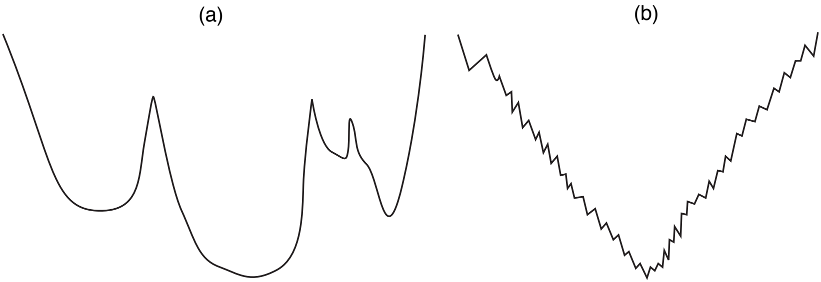

- Which method would most easily find the global minimum?
- What happens in hundreds or thousands of dimensions?
- What if different parts of the search space have different structure?

Clicker Question

Which of the following is true:

- A If an algorithm is above and to the left of another algorithm in a runtime distribution, it is always faster
- B A random walk cannot escape a local minima
- C The amount of time taken per step is about the same for all local search methods given modern data structures and the speed of computers
- D Carrying out arc consistency before doing a local search can reduce the search space