Assignment Nine: Optimizing Sequential Decisions

Solutions

Question One

(a) All of the random variables except for *Caught*1.

(b)

Cheat1	Caught1	Cheat2	value
T	T	T	14.09
T	T	F	14.59

(c) Cheat2 is a function of Cheat1 and Caught1:

Cheat1	Caught1	Cheat2
T	T	F

(d)

Cheat1	Caught1	value
T	T	14.59

(e) *Caught*1

(f) $Cheat1 = T$

(g) It is a factor of no variables, namely the number 81.69

(h) $83.18 - 81.69 = 1.49$

(i) $83.97 - 81.69 = 2.28$

Question Two

See `https://www.cs.ubc.ca/~poole/cs322/2020/as9/decChain.xml` for AISpace representation.

(a) For each i would add arcs from A_j and S_j to A_i for all $j < i$.

(b) There is a unique ordering:

Variable Eliminated	How	Factors Removed	Factor added	Decision Function
S_3	sum	$P(S_3 \mid S_2, A_2)\ V(S_3)$	$Q(S_2, A_2)$	
A_2	max	$Q(S_2, A_2)$	$V(S_2)$	$D_2(S_2)$
S_2	sum	$P(S_2 \mid S_1, A_1)\ V(S_2)$	$Q(S_1, A_1)$	
A_1	max	$Q(S_1, A_1)$	$V(S_1)$	$D_1(S_1)$
S_1	sum	$P(S_1 \mid S_0, A_0)\ V(S_1)$	$Q(S_0, A_0)$	
A_0	max	$Q(S_0, A_0)$	$V(S_0)$	$D_0(S_0)$
S_0	sum	$P(S_0)\ V(S_0)$	E	

where E is the expected value.

(c) At the first decision, reset on 4, hold on 3, otherwise flip.

At the second decision, same as the first decision, except reset on 2.

At the 3rd decision, same as the first decision, except hold on 2.

(d) It does not affect the optimal policy (it is not even considered when optimizing the policy), but affects the expected utility.

E.g, if it starts at 3 (with probability 1), it always holds with a value of 10. If it starts at 4 (with probability 1), it has a value of 6.65. (These can be seen in $V(S_0)$.)

(e) At each stage it has to do a sum and a max.

The sum uses time $O(s^2 d)$ (which is the size of the probability), and space (sd) to store the resulting factor, but the factor does not need to be kept, and so can be discarded after the max (and so the space does not need to be multiplied by n).

The max takes time $O(sd)$, and creates a factor of size s, and a decision function of size s. The decision functions need to be stored forever, so it uses space $O(sn)$.

So overall the time complexity $O(s^2 dn)$. The space complexity is $O(sd + sn)$

Question Three

It should not have taken more than a few hours. Most of this should have been in understanding the material and playing, not in doing busy work. I hope it was reasonable, and you learned something.