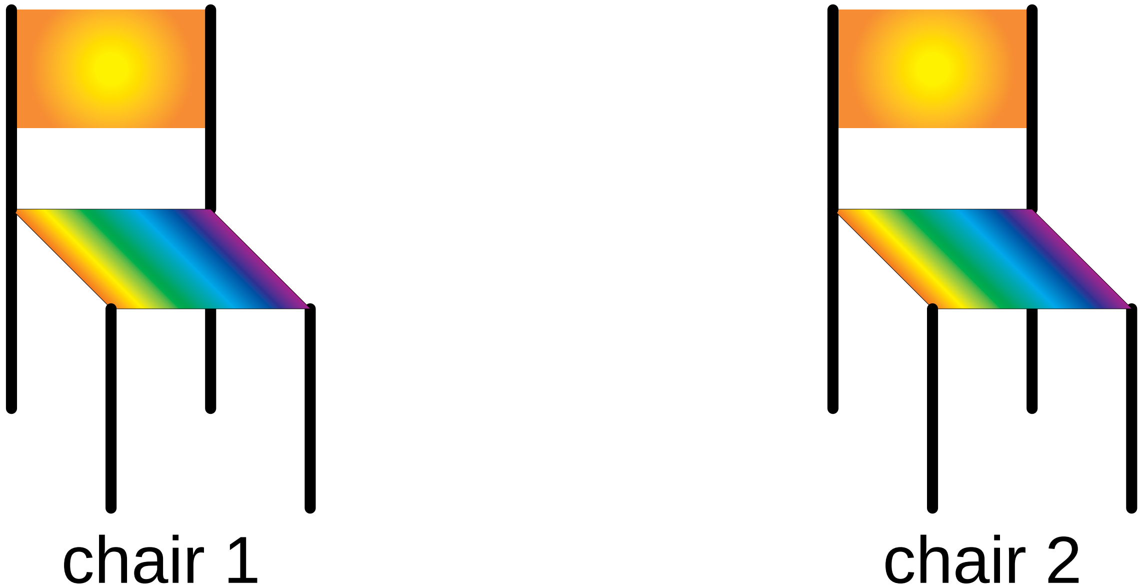
Equality

- Sometimes two terms denote the same individual.
- Example: Clark Kent & superman. 4×4 & $11 + 5$. The projector we used last Friday & this projector.
- Ground term t_1 **equals** ground term t_2, written $t_1 = t_2$, is true in interpretation I if t_1 and t_2 denote the same individual in interpretation I.

Equality doesn't mean similarity



$chair1 \neq chair2$

$chair\_on\_right = chair2$

$chair\_on\_right$ is not similar to $chair2$, it is $chair2$.

Why is equality important?

- In a doctor's office, the doctor wants to know if a patient is the same patient that she saw last week (or is his twin sister).
- In a criminal investigation, the police want to determine if someone is the same person as the person who committed some crime.
- When buying a replacement switch, an electrician may want to know if it was built in the same factory as the switches that were unreliable. (And if it is a different switch to the one that was replaced the previous time).

Allowing Equality Assertions

- Without equality assertions, the only thing that is equal to a ground term is itself.

  This can be captured as though you had the assertion $X = X$. Explicit equality never needs to be used.

- If you allow equality assertions, you need to derive what follows from them. Either:
  - axiomatize equality like any other predicate
  - build special-purpose inference machinery for equality

Axiomatizing Equality

$$X = X.$$

$$X = Y \leftarrow Y = X.$$

$$X = Z \leftarrow X = Y \wedge Y = Z.$$

For each n-ary function symbol f there is a rule of the form

$$f(X_1, \ldots, X_n) = f(Y_1, \ldots, Y_n) \leftarrow$$
$$X_1 = Y_1 \wedge \cdots \wedge X_n = Y_n.$$

For each n-ary predicate symbol p, there is a rule of the form

$$p(X_1, \ldots, X_n) \leftarrow$$
$$p(Y_1, \ldots, Y_n) \wedge X_1 = Y_1 \wedge \cdots \wedge X_n = Y_n.$$

Special-Purpose Equality Reasoning

paramodulation: if you have $t_1 = t_2$, then you can replace any occurrence of t_1 by t_2.

Treat equality as a **rewrite rule**, substituting equals for equals.

You select a **canonical representation** for each individual and rewrite all other representations into that representation.

Example: treat the sequence of digits as the canonical representation of the number.

Example: use the student number as the canonical representation for students.

Unique Names Assumption

The convention that different ground terms denote different individuals is the unique names assumption.

For every pair of distinct ground terms t_1 and t_2, assume $t_1 \neq t_2$, where "$\neq$" means "not equal to."

Example: For each pair of courses, you don't want to have to state, $math302 \neq psyc303$, …

Example: Sometimes the unique names assumption is inappropriate, for example $3 + 7 \neq 2 \times 5$ is wrong.

Axiomatizing Inequality for the UNA

- $c \neq c'$ for any distinct constants c and c'.
- $f(X_1, \ldots, X_n) \neq g(Y_1, \ldots, Y_m)$ for any distinct function symbols f and g.
- $f(X_1, \ldots, X_n) \neq f(Y_1, \ldots, Y_n) \leftarrow X_i \neq Y_i$, for any function symbol f. There are n instances of this schema for every n-ary function symbol f (one for each i such that $1 \leq i \leq n$).
- $f(X_1, \ldots, X_n) \neq c$ for any function symbol f and constant c.
- $t \neq X$ for any term t in which X appears (where t is not the term X).

Top-down procedure and the UNA

- Inequality isn't just another predicate. There are infinitely many answers to $X \neq f(Y)$.
- If you have a subgoal $t_1 \neq t_2$, for terms t_1 and t_2 there are three cases:
  - t_1 and t_2 don't unify. In this case, $t_1 \neq t_2$ succeeds.
  - t_1 and t_2 are identical including having the same variables in the same positions. Here $t_1 \neq t_2$ fails.
  - Otherwise, there are instances of $t_1 \neq t_2$ that succeed and instances of $t_1 \neq t_2$ that fail.

Implementing the UNA

- Recall: in SLD resolution you can select any subgoal in the body of an answer clause to solve next.

- Idea: only select inequality when it will either succeed or fail, otherwise select another subgoal. Thus you are delaying inequality goals.

- If only inequality subgoals remain, and none fail, the query succeeds.

Inequality Example

$$notin(X, []).$$

$$notin(X, [H|T]) \leftarrow X \neq H \wedge notin(X, T).$$

$$good\_course(C) \leftarrow course(C) \wedge passes\_analysis(C).$$

$$course(cs312).$$

$$course(cs444).$$

$$course(cs322).$$

$$passes\_analysis(C) \leftarrow something\_complicated(C).$$

$$?notin(C, [cs312, cs322, cs422, cs310, cs402])$$
$$\wedge good\_course(C).$$