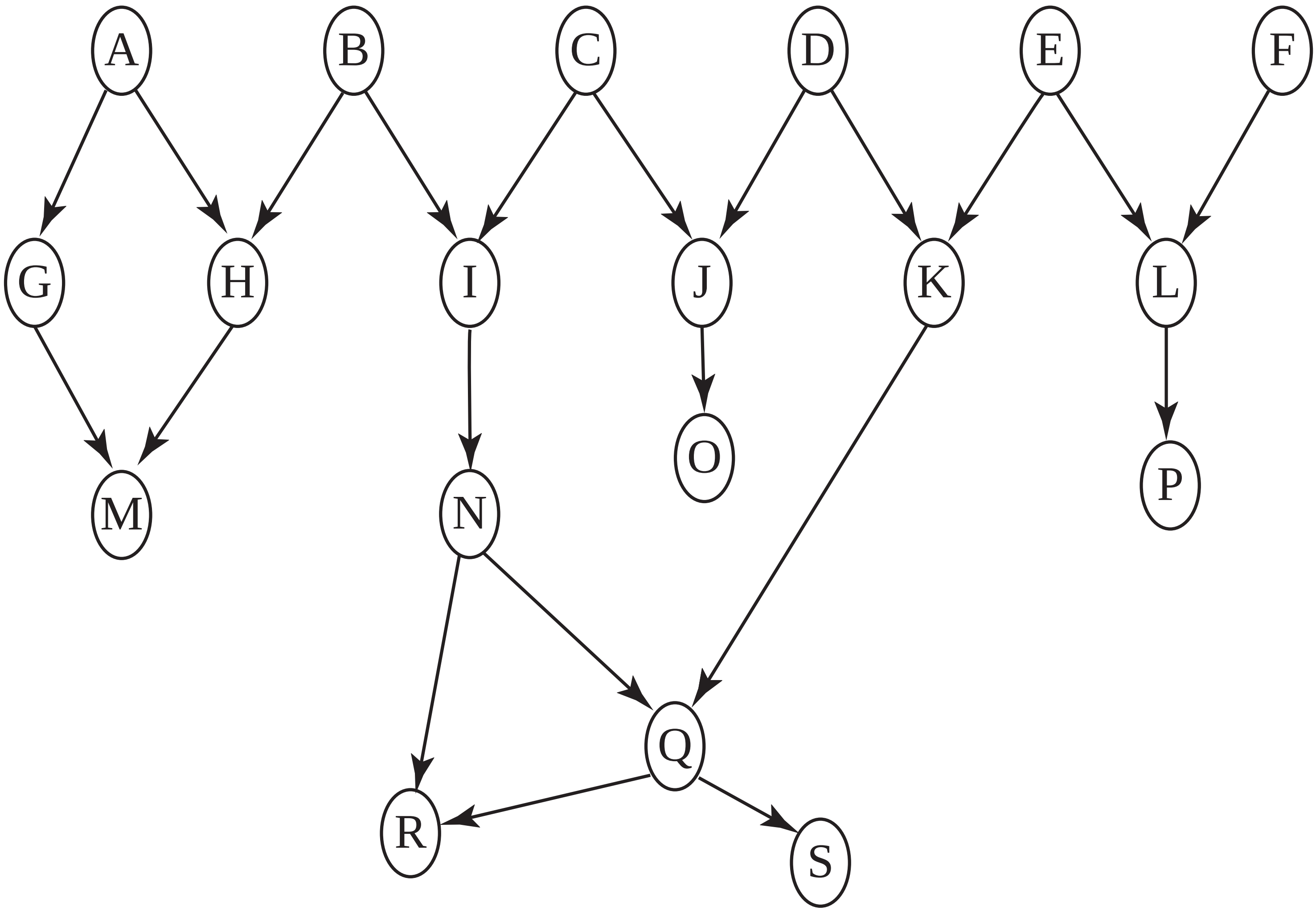
Understanding independence: example



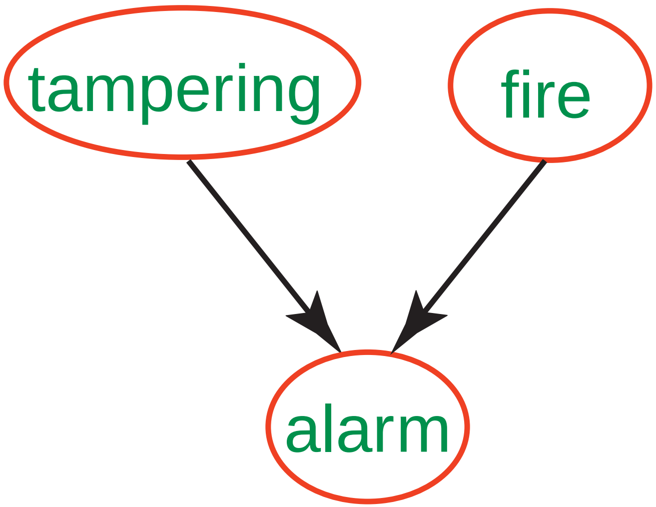
Understanding independence: questions

- On which given probabilities does $P(N)$ depend?
- If you were to observe a value for B, which variables' probabilities will change?
- If you were to observe a value for N, which variables' probabilities will change?
- Suppose you had observed a value for M; if you were to then observe a value for N, which variables' probabilities will change?
- Suppose you had observed B and Q; which variables' probabilities will change when you observe N?

What variables are affected by observing?

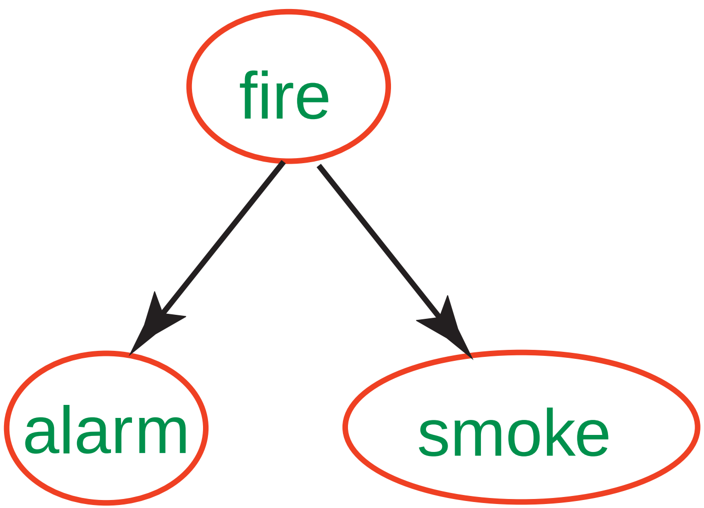
- If you observe variable $\overline{Y}$, the variables whose posterior probability is different from their prior are:
  - The ancestors of $\overline{Y}$ and
  - their descendants.
- Intuitively (if you have a causal belief network):
  - You do abduction to possible causes and
  - prediction from the causes.

Common descendants



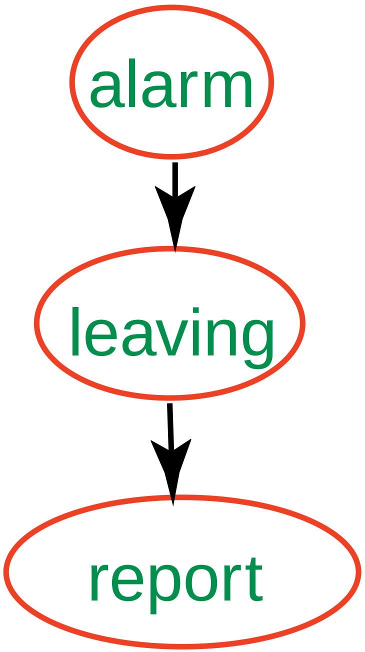
- *tampering* and *fire* are independent
- *tampering* and *fire* are dependent given *alarm*
- Intuitively, *tampering* can explain away *fire*

Common ancestors



- *alarm* and *smoke* are dependent
- *alarm* and *smoke* are independent given *fire*
- Intuitively, *fire* can explain *alarm* and *smoke*; learning one can affect the other by changing your belief in *fire*.

Chain



- *alarm* and *report* are dependent
- *alarm* and *report* are independent given *leaving*
- Intuitively, the only way that the *alarm* affects *report* is by affecting *leaving*.

d-separation

- $\overline{X}$ is d-separated from $\overline{Y}$ given $\overline{Z}$ if there is no path from an element of $\overline{X}$ to an element of $\overline{Y}$, where:
  - If there are paths $A \rightarrow B$ and $B \rightarrow C$ such that $B \notin \overline{Z}$, there is a path $A \rightarrow C$.
  - If there are paths $B \rightarrow A$ and $B \rightarrow C$ such that $B \notin \overline{Z}$, there is a path $A \rightarrow C$.
  - If there are paths $A \rightarrow B$ and $C \rightarrow B$ such that $B \in \overline{Z}$, there is a path $A \rightarrow C$.
- $\overline{X}$ is independent $\overline{Y}$ given $\overline{Z}$ for some conditional probabilities iff $\overline{X}$ is d-separated from $\overline{Y}$ given $\overline{Z}$