CPSC 320: Intermediate Algorithm Design and Analysis
Assignment #6, due Wednesday, June 17th, 2015 at 2:15pm in Room x235, Box 2

[8] 1. Consider a variant of Skip Lists in which nodes decide their height by throwing a six-sided die instead of a coin. If the die takes any of the values 2 through 6, then the node promotes itself to the next level. The node finalizes its level when the die roll is 1. For example, if a node rolls values 3, 5, 2, 1, then it has height 3 (it joins four lists). If a node rolls a 1 on the first trial, then it has height 0 (it joins only the bottom list).

Prove that, with this Skip List variant, if there are n nodes then the number of lists $O(\log n)$ with probability at least $1 - 1/n$.

[9] 2. This problem is about reductions. The textbook uses the notation $Y \leq_P X$ to denote that problem Y has a polynomial-time reduction to problem X. In other words, an arbitrary instance of problem Y can be solved using polynomially many calls to a subroutine that solves X, together with polynomially many additional computations.

Consider the following two problems, both of which we discussed in class:

- A: The Interval Scheduling Problem
- B: The Weighted Interval Scheduling Problem
- C: The Shortest Path Problem Problem
- D: The Longest Path Problem Problem

Remark: The Longest Path Problem is not discussed in the textbook, but it was mentioned in the lectures. It is an NP-complete problem.

[3] a. Is it true that $A \leq_P B$? Explain.

[3] b. Is it true that $B \leq_P A$? Explain.

[3] c. Is it true that $D \leq_P C$? Explain.

[10] 3. Consider the following computational problem.

- The CS department needs a committee to select the department's head. The committee cannot include people who have "conflicts of interest" with each other. The input consists of (a) the desired committee size, (b) a list of all the professors, and (c) a list of all pairs of professors that are conflicted. The goal is to determine whether there's a conflict-free committee of that size.

Remarks: For this question, it is important to understand the definition of NP (in Section 8.3), and the definitions of some of the standard NP-complete problems (Vertex Cover, Set Cover, Independent Set, Hamiltonian Path, etc.)

[5] a. Prove that this problem is in NP.

[5] b. Prove that this problem is NP-complete.

[1] 4. (Bonus) How long did it take you to complete this assignment (not including any time you spent revising your notes before starting)?