

CPSC 320: Intermediate Algorithm Design and Analysis  
Assignment #5, due Friday, June 12<sup>th</sup>, 2015 at 2:15pm in Room x235, Box 2

- [8] 1. A sequence of operations is performed on a stack whose size never exceeds  $k$ . After every  $k$  operations, a copy of the entire stack is made for backup purposes. Using the potential method, show that the cost of a sequence of  $n$  stack operations, including copying the stack, is in  $O(n)$ .
- [12] 2. Suppose that we want to implement a dynamic, open-address hash table. Recall that the load factor  $\alpha$  is defined by:

$$\alpha = n/s$$

where  $n$  is the number of elements in the hash table, and  $s$  is the size of the array used to store these elements. One possible scheme to make sure that the array is neither too big nor too small is the following:

- If an insertion causes  $\alpha$  to become greater than or equal to 90%, then we allocate a new array whose size is 1.5 times the size of the old array, and copy all of the elements to the new array.
- If a deletion causes  $\alpha$  to become less than or equal to 40%, then we allocate a new array whose size is 2/3rd the size of the old array, and copy all of the elements to the new array.

Use the potential method to prove that every sequence of  $t$  operations on an initially empty hash table using this scheme runs in  $O(t)$  time. You may assume the following:

- inserting or deleting an element (without reallocating the array) takes  $\Theta(1)$  time.
- creating a new array  $A'$  to replace an old array  $A$ , and copying all of the elements of  $A$  into  $A'$ , takes in  $\Theta(\max\{\text{length}(A), \text{length}(A')\})$ .

Hints: use  $\Phi(D_i) = 5n_i$ , where  $n_i$  is the number of operations that were performed on the data structure since the last time the array was reallocated. When you compute the amortized cost of an insertion or deletion that causes the array to be reallocated, determine the amortized cost of the insertion or deletion, and the amortized cost of the reallocation operation, separately.

- [1] 3. (Bonus) How long did it take you to complete this assignment (not including any time you spent revising your notes before starting)?