1. Let $L$ be a regular language. Let $L' \subseteq L$. Is $L'$ necessarily regular? Why?

2. A language $L$ is called finite if it contains finitely many strings. Prove that every finite language is regular.

3. Find regular expressions for the following languages.
   (a) \{ $w$ : $w$ contains the substring 0101 \}
   (b) \{ $w$ : $w$ has length at least 3 and its third symbol is a 0 \}
   (c) \{ $w$ : $w$ starts with a 0 and has odd length, or starts with a 1 and has even length \}
   (d) \{ $w$ : $w$ is any string except 11 and 111 \}

4. Can the following sets of strings be accepted by finite automata? Justify your answers!
   (a) \{ $1^n$ : $n$ is a prime number \};
   (b) \{ $0^{2n}1^{2m}$ : $n$ and $m$ are integers \};
   (c) \{ $x$ : $x$ is a binary power of two \};
   (d) \{ $x$ : the center symbol of $x$ is a 1 \}.

5. (A Worked Problem). Let $S$ be a language. Show that $S^* = (S^*)^*$.
   **Solution:** We need to show that $S^* \subseteq (S^*)^*$ and $(S^*)^* \subseteq S^*$. To show that $S^* \subseteq (S^*)^*$, recall that any word in a set is also in the set’s Kleene closure. For the other inclusion, suppose that $w \in (S^*)^*$. Then
   $$w = w_1 \cdots w_n$$
   for some $n \geq 0$, where $w_i \in S^*$ for all $i \in \{1, \ldots, n\}$. Since $w_i \in S^*$ for every $i$, it follows that
   $$w_i = w_{i,1} \cdots w_{i,k_i}$$
   for some $k_i \geq 0$, where $w_{i,j} \in S$ for all $j \in \{1, \ldots, k_i\}$. Then
   $$w = w_{1,1} \cdots w_{1,k_1} w_{2,1} \cdots w_{2,k_2} \cdots w_{n,1} \cdots w_{n,k_n},$$
   where $w_{i,j} \in S$. That is, $w$ is the concatenation of a finite number of words ($k_1 + \cdots + k_n$ words) in $S$, so by definition of the star operation, $w \in S^*$.

6. Show that the regular sets are not closed under infinite union by producing an infinite family of regular sets whose union is not regular.

7. An epsilon move takes place when a finite automaton reads and changes state but does not move its tape head. Does this new operation add power to finite automata? Justify your answer.
8. Let $\Sigma = \{0, 1, +, =\}$ and

$\text{ADD} = \{x = y + z : x, y, z \text{ are binary integers, and } x \text{ is the sum of } y \text{ and } z\}.$

Show that $\text{ADD}$ is not regular.

9. We can use closure properties to help prove certain languages are not regular. Start with the fact that the language

$L_{0^n1^n} = \{0^n1^n : n \geq 0\}$

is not regular. Prove the following languages not to be regular by transforming them, using operations known to preserve regularity, to $L_{0^n1^n}$:

(a) $\{0^i1^j : i \neq j\};$

(b) $\{0^n1^m2^{n-m} : n \geq m \geq 0\}.$