CPSC 421: Introduction to Theory of Computing
Practice Problem Set #1, Not to be handed in

1. Let L be a regular language. Let $L' \subseteq L$. Is L' necessarily regular? Why?

2. A language L is called *finite* if it contains finitely many strings. Prove that every finite language is regular.

3. Find regular expressions for the following languages.

   (a) $\{ w : w$ contains the substring 0101 $\}$

   (b) $\{ w : w$ has length at least 3 and its third symbol is a 0 $\}$

   (c) $\{ w : w$ starts with a 0 and has odd length, or starts with a 1 and has even length $\}$

   (d) $\{ w : w$ is any string except 11 and 111 $\}$

4. Can the following sets of strings be accepted by finite automata? Justify your answers!

   (a) $\{1^n : n$ is a prime number$\}$;

   (b) $\{0^{2n}1^{2m} : n$ and m are integers$\}$;

   (c) $\{x : x$ is a binary power of two$\}$;

   (d) $\{x :$ the center symbol of x is a 1$\}$.

5. (A Worked Problem). Let S be a language. Show that $S^* = (S^*)^*$.
   Solution: We need to show that $S^* \subseteq (S^*)^*$ and $(S^*)^* \subseteq S^*$. To show that $S^* \subseteq (S^*)^*$, recall that any word in a set is also in the set's Kleene closure. For the other inclusion, suppose that $w \in (S^*)^*$. Then

   $$w = w_1 \cdots w_n$$

   for some $n \geq 0$, where $w_i \in S^*$ for all $i \in \{1, \ldots, n\}$. Since $w_i \in S^*$ for every i, it follows that

   $$w_i = w_{i,1} \cdots w_{i,k_i}$$

   for some $k_i \geq 0$, where $w_{i,j} \in S$ for all $j \in \{1, \ldots, k_i\}$. Then

   $$w = w_{1,1} \cdots w_{1,k_1} w_2 \cdots w_{2,k_2} \cdots w_n \cdots w_{n,k_n},$$

   where $w_{i,j} \in S$. That is, w is the concatenation of a finite number of words ($k_1 + \cdots + k_n$ words) in S, so by definition of the star operation, $w \in S^*$.

6. Show that the regular sets are not closed under infinite union by producing an infinite family of regular sets whose union is not regular.

7. An *epsilon move* takes place when a finite automaton reads and changes state but does not move its tape head. Does this new operation add power to finite automata? Justify your answer.

8. Let $\Sigma = \{0, 1, +, =\}$ and

$\texttt{ADD} = \{x = y + z : x, y, z$ are binary integers, and x is the sum of y and $z\}$.

Show that `ADD` is not regular.

9. We can use closure properties to help prove certain languages are not regular. Start with the fact that the language

$L_{0n1n} = \{0^n 1^n : n \geq 0\}$

is not regular. Prove the following languages not to be regular by transforming them, using operations known to preserve regularity, to L_{0n1n}:

   (a) $\{0^i 1^j : i \neq j\}$;
   (b) $\{0^n 1^m 2^{n-m} : n \geq m \geq 0\}$.