

CPSC 421: Introduction to Theory of Computing  
Assignment #5, due Thursday October 13th by 11:59pm, via Gradescope

- [5] 1. Describe a TM that decides the language.

$$L_1 = \{ a^n b^n c^n : n \geq 0 \}.$$

Your description should be a *high-level description* wherein, according to Sipser (page 185),

we use English prose to describe an algorithm, ignoring the implementation details.  
At this level we do not need to mention how the machine manages its tape or head.

You may use multiple tapes or non-determinism if you wish.

- [6] 2. Suppose that  $L_1$  and  $L_2$  are recognizable languages. Describe a Turing machine that recognizes

$$L_1 \circ L_2 = \{ xy : x \in L_1, y \in L_2 \}.$$

You may use multiple tapes or non-determinism if you wish.

- [10] 3. A **high speed** Turing machine is similar to an ordinary deterministic TM but with a modified transition function. For any state  $q \in Q$  and any symbol  $a \in \Gamma$ , the transition function defines

$$\delta(q, a) = (p, b, x)$$

where  $p \in Q$  is the new state,  $b \in \Gamma$  is the new symbol written on the tape, and  $x \in \mathbb{Z}$  is an integer specifying how many positions to move the tape head. If  $x$  is positive, it moves to the right by  $x$  positions; if  $x$  is negative, it moves to the left by  $|x|$  positions.

As usual, if the tape head moves off the left end of the tape, the TM “crashes” by moving to the reject state.

Show that for every high speed TM  $M$ , there exists an ordinary deterministic TM  $M'$  for which  $L(M') = L(M)$ . If you like,  $M'$  may have multiple tapes.

- [5] 4. Describe a **nondeterministic** Turing machine to recognize the following language:

$$L_2 = \{ \langle M \rangle : M \text{ is a TM and } M \text{ halts on some input} \}.$$

- [5] 5. Fix the alphabet  $\Sigma = \{0, 1\}$ . Let  $\mathcal{A}$  be the class of languages whose cardinality is a power of two. That is,

$$\mathcal{A} = \{ L \subseteq \Sigma^* : \exists n \in \mathbb{N} \text{ s.t. } |L| = 2^n \}.$$

Is  $\mathcal{A}$  countable or not? Justify your answer.

**Hints:**

- This can be answered in just a couple of sentences.
- It follows from some facts discussed in the lectures.