CPSC 421: Introduction to Theory of Computing
Assignment #4, due October 4th by 11:59pm, via GradeScope.com

[15] 1. [10] a. Give a context-free grammar that generates the language

$$L = \left\{ w^i x^j y^k z^\ell \ : \ i, j, k, \ell \geq 0 \text{ such that } j \leq 3 \text{ and } (i = k \text{ or } i = \ell) \right\}.$$

You should explain why your grammar works..

[5] b. Is your grammar ambiguous? Why or why not?

[8] 2. The following are true/false questions. **Justify your answers**, in about one sentence.

[2] a. There exists a Turing Machine such that $|Q| = 1$.

[2] b. There exist a decidable language which is not recognizable.

[2] c. Let A be a decidable language (assume $A \neq \Sigma^*$). If M is a Turing machine that recognizes A, can it be the case that M runs forever for some inputs $w \notin A$?
(More precisely, does there exist M and w such that $L(M) = A$, $w \notin A$ and M runs forever on input w?)

[2] d. Let A be a decidable language (assume $A \neq \Sigma^*$). If M is a Turing machine that decides A, can it be the case that M runs forever for some inputs $w \in A$?
(More precisely, does there exist M and w such that $L(M) = A$, M is a decider, $w \in A$ and M runs forever on input w?)

[10] 3. [5] a. Show that the class of decidable languages is closed under complement (i.e., if L is decidable then $\Sigma^* \setminus L$ is decidable).

[5] b. Does the same argument as part (a) show that the class of Turing-recognizable languages is closed under complement? Why or why not?

[6] 4. Describe a Turing Machine which decides the following language

$$L \ = \ \{ \ x_1\#x_2\#x_3\#x_4 \ : \ x_k \in \{0,1\}^* \ \forall k, \ x_2 = x_3, \ x_2 \neq x_4 \}.$$

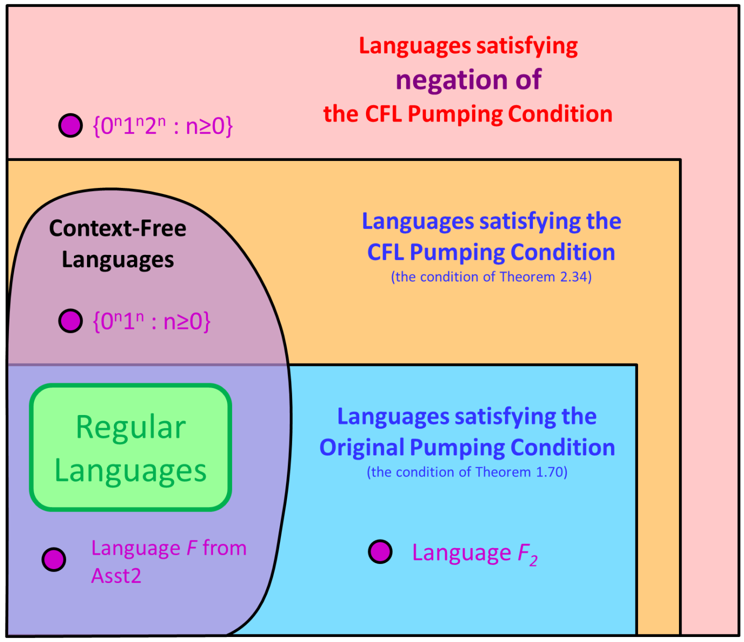
Your description should be an *implementation description* which, according to Sipser (page 185), is

> a higher level of description in which we use English prose to describe the way that the Turing machine moves its head and the way that it stores data on its tape.

So there is no need to explicitly describe the transition function, states, etc., of the Turing Machine.

You should use a single-tape, deterministic Turing machine.

[1] 5. **OPTIONAL BONUS QUESTION:** The purpose of this question is basically to prove that the following diagram is correct.



[1] a. Prove that any language satisfying the **Original** Pumping Condition (i.e., the condition of Theorem 1.70) also satisfies the **CFL** Pumping Condition (i.e., the condition of Theorem 2.34).

[2] b. Prove that there is an alphabet Σ and a language F_2 over Σ that statisfies the **Original** Pumping Condition (i.e., the condition of Theorem 1.70), but is **not** context-free.