CPSC 421: Introduction to Theory of Computing
Assignment #3, due Tuesday September 27th by 11:59pm, via GradeScope.ca

[4] 1. [2] a. Prove that if L_1 and L_2 are regular, then so is

$$L_1 \setminus L_2 \;=\; \{\, x \in \Sigma^* \,:\, x \in L_1 \text{ and } x \notin L_2 \,\}.$$

You may wish to use results from the textbook (e.g., pages 46 and 85). (My solution is one sentence long.)

[2] b. Let k be a fixed positive integer. Define $L_k = \left\{\, 0^{kn} 1^{km} \,:\, n, m \geq 0 \,\right\}$. Is L_k regular? Briefly explain why or why not.

[6] 2. Let R be the regular expression $\big((1^*01) \cup (00)\big)^* 10^* \big((0^*01) \cup \epsilon\big)$. Let $L(R)$ be the language that it generates.

The following are true/false questions, and you do not need to justify your answers.

[2] a. Is $010 \in L(R)$?

[2] b. Is $10100 \in L(R)$?

[2] c. Is $001001 \in L(R)$?

[8] 3. Let $\Sigma = \{a, b\}$ and $L = \{\, w \in \Sigma^* \,:\, w \text{ is a palindrome } \}$. (Recall that a palindrome is any string x that equals the reverse of itself.) In this question we will prove that L is non-regular using the Pumping Lemma.

[1] a. State the negation of the pumping condition.

[3] b. For every integer p, define a string w (of length at least p) that is a palindrome over Σ and for which the first p characters are identical. (But w should not be of the form a^* or b^*).

[3] c. Show that for any decomposition of your palindrome w into xyz where $|y| > 0$ and $|xy| \leq p$, that xy^2z is no longer a palindrome.

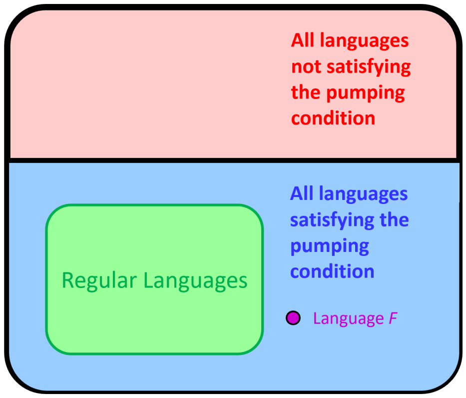
[1] d. Conclude that L is not regular.

[8] 4. Let $\Sigma = \{a, b, c, d\}$ and $L = \left\{\, a^7 b^n c^3 d^{5n} \,:\, n \geq 0 \,\right\}$. Prove that L is non-regular using the Pumping Lemma.

[15] 5. Let $\Sigma = \{0, 1, 2\}$. Consider the language

$$F \;=\; \{\, 01^n 2^n \,:\, n \geq 1 \,\} \cup \left\{\, 0^k w \,:\, k \neq 1,\; w \in \{1,2\}^* \,\right\}.$$

The purpose of this question is to prove that F is non-regular, but it satisfies the pumping condition. In other words, the situation looks like this figure:



[7] a. Show that F is not regular. You may use the pumping lemma and the fact that the class of regular languages is closed under unions, concatenations and complements. You may assume that

$$\{ 01^n2^n : n \geq 0 \}$$

is non-regular since we have shown something very similar in class.

[7] b. Show that F satisfies the pumping condition. In other words, find an appropriate integer p and demonstrate that for all strings $w \in F$ with $|w| \geq p$, we can decompose $w = xyz$ (where $x, y, z \in \{0, 1, 2\}^*$) such that

- $|y| > 0$,
- $|xy| \leq p$, and
- $xy^nz \in F$ for all $n \geq 0$.

[1] c. Explain why this does not contradict the pumping lemma.

[10] 6. Let $\Sigma = \{a, b\}$. Let

$$L = \{ x \in \Sigma^* : x \text{ is a palindrome and } |x| \text{ is a multiple of } 3 \}.$$

[5] a. Give a PDA that recognizes L.

[5] b. Give a CFG that generates L.

[2] 7. **OPTIONAL BONUS QUESTION**

Professor Dumas thinks he understands the pumping lemma. His interpretation is that "a finite automaton has a limited amount of complexity, so if it accepts a very long string, that string must have some repeated structure inside". To formalize his interpretation, he proposes the following conjecture.

Conjecture 1. *Let L be regular language. Suppose there is a DFA with p states that accepts L. Then for every string $w \in L$ with $|w| \geq p$,*

$$\exists x, y, z \in \Sigma^* \text{ and } i \geq 2 \quad \text{such that} \quad w = xy^iz \text{ and } y \neq \epsilon. \tag{1}$$

Professor Dumas' conjecture is false. Let's try to understand why.

Let $\Sigma = \{a, b, c\}$. Find a DFA $M = (Q, \Sigma, \delta, q_0, F)$ and a string w that is accepted by M, and has $|w| > 20|Q|$, but does not satisfy (1).