CPSC 421: Introduction to Theory of Computing
Assignment #3, due Friday Oct 5th by 3:00pm, via GradeScope.com

[1] 1. **LATEX BONUS!** You get 1 bonus mark if the homework is typeset using Latex.

[15] 2. [10] a. Give a context-free grammar that generates the language

$$L = \left\{ w^i x^j y^k z^\ell \ : \ i, j, k, \ell \geq 0 \text{ such that } j \leq 3 \text{ and } (i = k \text{ or } i = \ell) \right\}.$$

You should explain why your grammar works..

[5] b. Is your grammar ambiguous? Why or why not?

[8] 3. The following are true/false questions. **Justify your answers**, in about one sentence.

[2] a. There exists a Turing Machine such that $|Q| = 1$.

[2] b. There exist a decidable language which is not recognizable.

[2] c. Suppose M recognizes L. Then, M can run forever for some inputs $w \notin L$.

[2] d. Suppose M decides L. Then, M can run forever for some inputs $w \in L$.

[10] 4. [5] a. Show that the class of decidable languages is closed under complement (i.e., if L is decidable then $\Sigma^* \setminus L$ is decidable).

[5] b. Does the same argument as part (a) show that the class of Turing-recognizable languages is closed under complement? Why or why not?

[6] 5. Describe a Turing Machine which decides the following language

$$\begin{aligned} L \ = \ \{ \ x_1\#x_2\# \ldots \#x_n \ : \ & n \text{ is even}, \ x_k \in \{0,1\}^* \ \forall k, \\ & x_{2k} = x_{2k+1} \ \forall k, \ x_{2k} \neq x_{2(k+1)} \ \forall k \}. \end{aligned}$$

Your description should be an *implementation description* which, according to Sipser (page 185), is

> a higher level of description in which we use English prose to describe the way that the Turing machine moves its head and the way that it stores data on its tape.

So there is no need to explicitly describe the transition function, states, etc., of the Turing Machine.

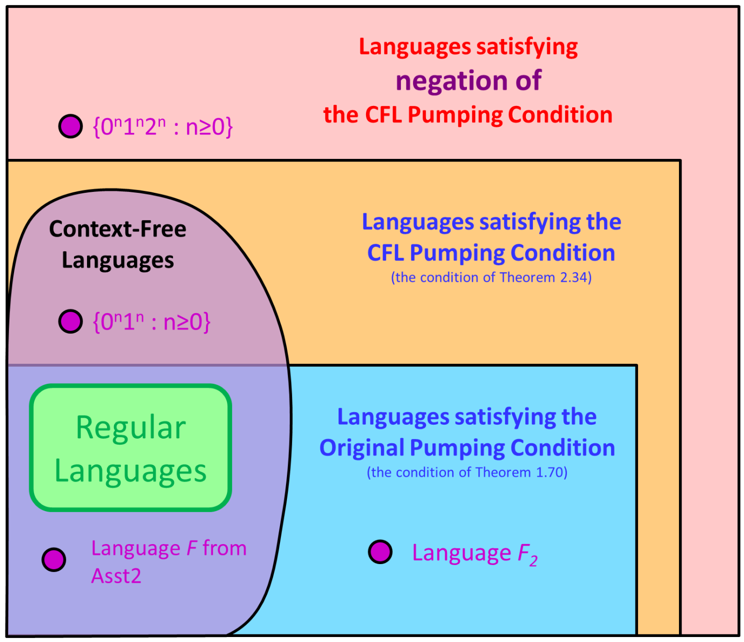
[6] 6. Suppose that L_1 and L_2 are Turing-recognizable. Describe a Turing machine that recognizes

$$L_1 \circ L_2 \ = \ \{ \ xy \ : \ x \in L_1, \ y \in L_2 \ \}.$$

Your description should be a *high-level description* wherein, according to Sipser (page 185),

> we use English prose to describe an algorithm, ignoring the implementation details. At this level we do not need to mention how the machine manages its tape or head.

[1] 7. **OPTIONAL BONUS QUESTION:** The purpose of this question is basically to prove that the following diagram is correct.



[1] a. Prove that any language satisfying the **Original** Pumping Condition (i.e., the condition of Theorem 1.70) also satisfies the **CFL** Pumping Condition (i.e., the condition of Theorem 2.34).

[2] b. Prove that there is an alphabet Σ and a language F_2 over Σ that statisfies the **Original** Pumping Condition (i.e., the condition of Theorem 1.70), but is **not** context-free.