1. Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, and $C = \{1, 5, 9\}$ be subsets of some universal set $U = \{0, 1, 2, ..., 9\}$. Determine:
   (a) $A \cap B$
   (b) $A \cup B$
   (c) $A \cup C$
   (d) $A \cap C$
   (e) $A - B$
   (f) $B - A$
   (g) $\overline{A}$
   (h) $\overline{C}$

2. Let $A = \{1, 2, 3\}$, $B = \{2, 3\}$, $C = \{1, 4\}$, and let the universal set be $U = \{0, 1, 2, 3, 4\}$. List the elements in:
   (a) $A \times B$
   (b) $B \times A$
   (c) $A \times B \times C$
   (d) $A \times \overline{A}$
   (e) $2^A$
   (f) $A \times \emptyset$
   (g) $B \times 2^B$

3. Let $A$, $B$, and $C$ be as in question 1 and let $D = \{3, 2\}$ and $E = \{2, 3, 2\}$. Determine which of the following are true. Give reasons for your decisions.
   (a) $A = B$
   (b) $B = D$
   (c) $B = E$
   (d) $A - B = B - A$

4. For sets $A$, $B$ and $C$, prove the following:
   (a) If $A \subseteq B$ then $A \cap B = A$
   (b) If $A \subseteq B$ then $A \cup B = B$
   (c) $(A \cup B) \times C = (A \times C) \cup (B \times C)$.

5. Prove that if $A \subseteq B$ and $C \subseteq D$, then $A \times C \subseteq B \times D$

6. Given that $U = \text{all university students}$, $D = \text{day students}$, $M = \text{mathematics majors}$, and $G = \text{graduate students}$. Draw Venn diagrams illustrating this situation and shade in the following sets.
(a) evening (i.e. non-day) students
(b) undergraduate mathematics majors
(c) non-math graduate students
(d) non-math undergraduate students
(e) graduate students or math majors who take day classes

7. Let $A$ be a set with $|A| = n$. How many distinct two-element subsets are there in the set $2^A$?

8. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{a, b, c, d, e, f\}$, and $C = \{+, -\}$. Define the functions $f : A \to B$ such that $f(k) =$ the $k^{th}$ letter in the alphabet, and $g : B \to C$ such that $g(\alpha) = +$ if $\alpha$ is a vowel and $g(\alpha) = -$ if $\alpha$ is a consonant.

(a) Find $g \circ f$
(b) Does it make sense to discuss $f \circ g$? If not, why not?
(c) Does $f^{-1}$ exist? Why or why not?
(d) Does $g^{-1}$ exist? Why or why not?

9. (*) For each of the following pairs of sets $X$ and $Y$, give a function $f : X \to Y$ that is:

(i) Injective but not surjective
(ii) Surjective but not injective
(iii) Bijective

Or else explain why this is not possible.

(a) $X = \{a, b\}$, $Y = \{1, 2, 3\}$
(b) $X = \{a, b, c\}$, $Y = \{1, 2\}$
(c) $X = \{a, b, c\}$, $Y = \{1, 2, 3\}$

10. (*) There is a theorem stating that two sets have the same cardinality (i.e., number of elements) if and only if there exists a bijection between them. This is rather trivial if the sets are finite, but can be a powerful tool for analyzing the sizes of infinite sets. Show that there exists a bijection from $\mathbb{N}$, the set of natural numbers ($\{0, 1, 2, \ldots\}$), to $\mathbb{Z}$, the set of integers ($\{\ldots, -2, -1, 0, 1, 2, \ldots\}$). Conclude that these two sets in fact have the same number of elements.

11. Let $A$ be an arbitrary set, and let $B = \{x \in A : x \notin x\}$. That is, $B$ contains all sets in $A$ that do not contain themselves: For all $y$,

 (*) $y \in B$ if and only if ($y \in A$ and $y \notin y$).

Can it be that $B \in A$?

12. Let $\mathcal{A}$ be a family of sets. The union of the sets in $\mathcal{A}$ is

$$\bigcup_{A \in \mathcal{A}} A = \{a : a \in A \text{ for some } A \in \mathcal{A}\},$$

and the intersection of the sets in $\mathcal{A}$ is

$$\bigcap_{A \in \mathcal{A}} A = \{a : a \in A \text{ for all } A \in \mathcal{A}\}.$$
(a) What is \( \bigcup_{A \in \emptyset} A \) ?

(b) What is \( \bigcap_{A \in \emptyset} A \)? (Which sets do not satisfy the definition of intersection?)

(c) Show that \( (\bigcup_{A \in \mathcal{A}} A)^c = \bigcap_{A \in \mathcal{A}} A^c \). Note that \( \mathcal{A} \) needn’t be countable; it is arbitrary.

13. Show that for any sets \( A \) and \( B \),

\[
A = (A \cap B) \cup (A \cap B^c) = ((A \cup B) \cap B^c) \cup (A \cap B).
\]