

**Surname (print):** \_\_\_\_\_

**First Name:** \_\_\_\_\_

**Signature:** \_\_\_\_\_

**ID #:** \_\_\_\_\_

University of British Columbia  
**CPSC 421/501 Introduction to Theory of Computation**  
Midterm Exam  
October 19, 2012  
8:00 a.m. - 8:50 a.m.

**INSTRUCTIONS:**

- 1: Write your name and ID# in the blanks above.
- 2: There are 6 pages in this exam, including the cover page and two blank pages at the back. Make sure that you have all the pages.
- 3: No textbooks, notes, cheatsheets or electronic devices are permitted.

Question	Value	Mark Awarded
1	20	
2	20	
3	20	
Total	60	

**Question 1:** True/False Questions. You do not need to justify your answer. You may assume that the alphabet is  $\{a, b\}$ .

- 1: If  $L_1$  and  $L_2$  are regular then  $L_1 \setminus L_2$  must be regular. ( $L_1 \setminus L_2$  is the set of strings that are in  $L_1$  but not in  $L_2$ .)
  
  
  
  
  
  
  
  
  
  
- 2: The language  $\{ ab^{3n}a : n \geq 0 \}$  is regular.
  
  
  
  
  
  
  
  
  
  
- 3: If  $L_1$  and  $L_2$  are context-free then  $L_1 \cap L_2$  must be context-free.
  
  
  
  
  
  
  
  
  
  
- 4: The language  $\{ w : |w| \text{ is odd and } w\text{'s middle symbol is } a \}$  is context-free.
  
  
  
  
  
  
  
  
  
  
- 5: If  $L_1$  and  $L_2$  are decidable languages then  $L_1 \cup L_2$  must be decidable.

**Question 2:** Consider the grammar  $G = (V, \Sigma, R, E)$  where  $V = \{E, T, F\}$  and  $\Sigma = \{1, 2, 3, +, \times, (, )\}$ . The rules are

$$\begin{aligned} E &\rightarrow E + T \mid T \\ T &\rightarrow T \times F \mid F \\ F &\rightarrow (E) \mid 1 \mid 2 \mid 3 \end{aligned}$$

Draw a parse tree for the string  $3 \times (1 + 2)$ .

**Question 3:**

(a): State the Pumping Lemma for regular languages.

(b): Use the Pumping Lemma to prove that the language  $L = \{ a^{2^n} : n \geq 0 \}$  is not regular. Here,  $a^{2^n}$  means a string of  $2^n$   $a$ 's. (**Hint:** Very few integers are powers of two.)

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