CPSC 421/501 Intro to Theory of Computing (Term 1, 2013-14)
Assignment 2

Due: Monday, September 30th in class.

Question 1: [10 points]
Let R and S be regular expressions over some alphabet Σ. Formally prove or disprove each of the following statements. (Here we use "=" to mean that the two regular expressions describe the same language.)

- **(a):** $(R^*S^*)^* = (R \cup S)^*$.
- **(b):** $R^* \cup S^* = (R \cup S)^*$
- **(c):** $(R \cup S)^*S = (R^*S)^*$.

Question 2: [10 points]
Prove that the following languages over $\Sigma = \{0, 1\}$ are not regular. You may use the fact that the class of regular languages is closed under unions, concatenations and complements.

- **(a):** $L = \{\ 0^n 1^m \ :\ n \leq m\ \}$.
- **(b):** $L = \{\ w \ :\ |w| \text{ is a perfect square } \}$. (That is, $|w| = n^2$ for some integer $n \geq 0$.)
- **(c):** $L = \{\ w \ :\ w \text{ is not a palindrome } \}$. (A palindrome is a string that reads the same forward and backward.)

Question 3: [10 points]
Consider the language $F = \left\{\ a^i b^j c^k \ :\ i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\ \right\}$.

- **(a):** Show that F is not regular.
- **(b):** Show that F satisfies the pumping condition. In other words, give an integer $p \geq 1$ and demonstrate that for all strings $w \in F$ with $|w| \geq p$ we can find $x, y, z \in \{a, b, c\}^*$ such that
  - $xy^n z \in F$ for all $n \geq 0$,
  - $|y| > 0$, and
  - $|xy| \leq p$.
- **(c):** Explain why this does not contradict the pumping lemma.

Question 4: [10 points]

Let $\Sigma = \{0, 1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words, let $B = \{\ uv \ :\ u \in \Sigma^*,\ v \in \Sigma^* 1 \Sigma^* \text{ and } |u| \geq |v|\ \}$.

- **(a):** Give a PDA that recognizes B.
- **(b):** Give a CFG that generates B.

Question 5: OPTIONAL BONUS QUESTION [20 points]

Professor Dumas thinks he understands the pumping lemma. His interpretation is that "a finite automaton has a limited amount of complexity, so if it accepts a very long string, that string must have some repeated structure inside". To formalize his interpretation, he proposes the following conjecture.

Conjecture 1. Let L be regular language over Σ. Then there exists an integer $p \geq 1$ such that, for every string $w \in L$ with $|w| \geq p$,

$$\exists x, y, z \in \Sigma^* \text{ and } i \geq 2 \quad \text{such that} \quad w = xy^i z \text{ and } y \neq \epsilon. \tag{1}$$

Professor Dumas' conjecture is false. Let's try to understand why.

- **(a):** Find a DFA $M = (Q, \Sigma, \delta, q_0, F)$ with with $|Q| \leq 3$ and $|\Sigma| \geq 3$ and a string w that is accepted by M, and has $|w| > |\Sigma|^2$, but does not satisfy (1).
- **(b):** Can you find an even longer string w that is accepted by your M but does not satisfy (1)? You can use computer simulations if you like. If your solution involves any ideas found in the literature or online, be sure to give a citation.