CPSC 421/501 Intro to Theory of Computing (Term 1, 2012-13)
Assignment 3

Due: Friday Nov 2nd, in class.

For any question (except the bonus question), if you write "I do not know the answer to this question", you will receive 20% of the marks for that question.

Question 1: Alan Turing's second graduate student, John Smartypants has designed a new machine that he thinks is more powerful than the Turing Machine. Whereas a Turing Machine has a linear tape of infinite length, the new machine has a three-dimensional memory arranged into a grid, that is infinite in every direction. At every step of the computation, the machine's head can move up, down, left, right, forward or backward. Prove that Smartypants' machine is not more powerful than a Turing machine.

(Your answer should have roughly the same level of detail as the proof of Theorem 3.13 in the text.)

Question 2: Exercise 4.18 (2nd edition)

Let A and B be two disjoint languages over the alphabet Σ. Say that language C **separates** A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-recognizable languages are separable by some decidable language. (A language A is said to be **co-recognizable** if its complement, namely $\overline{A}$, is recognizable.)

Question 3: Define the language L_i by

$$L_i \;=\; \left\{ \langle\, M, w \,\rangle \,:\, \text{the TM } M \text{ on input } w \text{ never writes to the } i^{\text{th}} \text{ square of its tape} \right\}.$$

Show that, for any $i \geq 1$, the language L_i is undecidable.

OPTIONAL BONUS QUESTION:

Question 4: In any programming language of your choice, write a program that outputs its own source code, except that all upper-case letters have been replaced with lower-case letters, and vice-versa.

For example, suppose I write the program in C and my file is called `program.c` and the corresponding executable is called `program`. Then the command

```
./program | perl -pe "tr/A-Za-z/a-zA-Z/;"
```

should produce the same output as the command

```
cat program.c
```

If you solve this question, please send the solution to Prof. Harvey by email.