A dynamic theory of social failure in isolated communities

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Keywords. Likelihood-based approach; Mixed membership analysis; Social networks.

Summary. We introduce a statistical model for dynamic network analysis and we apply it to Sampson's longitudinal collection of the sociometric relation *trust*, among 18 novices in a monastery (Sampson, 1968). A preliminary analysis offers a suggestive characterization of the social dynamics that led to the collapse of the congregation—purportedly over religious differences. Such a characterization provides the elements for a theory of failure in isolated communities that is rooted in latent aspects of the dynamics of social interaction. Namely, the existence of tight informal groups as a precondition, the progressive polarization of such groups as a signal of increasing differences, and the emergence of an interstitial group of individuals over time as a signal of imminent conflict.

Problem. Sampson spent several months in a monastery in New England, where a number of novices were preparing to join a monastic order. He set out to measure sociometric relations among them by repeatedly administering a questionnaire, and determined asymmetric social relationships within the group. Sampson's original analysis was rooted in direct anthropological observations. He strongly suggested the existence of tight factions among the novices: the loyal opposition (whose members joined the monastery first), the young turks (who joined later on), the outcasts (who were not accepted in the two main factions), and the waverers (who did not take sides). The events that took place during Sampson's stay at the monastery supported his observations; members of the young turks resigned after their leaders were expelled over religious differences—the novices John Bosco and Gregory in Figure 1, points no. 9 and 17, respectively. Faction labels assigned by Sampson to the novices are commonly regarded as *true labels* in the literature. Analyses of a static collation (Breiger et al., 1975) of this classic data set abound (Fienberg et al., 1985; Hoff et al., 2002; Davis and Carley, 2006; Airoldi et al., 2007; Handcock et al., 2007, with discussion).

Here, we focus on the responses to the question "who do you trust?", which was repeated at three subsequent epochs—novices were requested to name up to three other members of the congregation.

Model. We introduce an explicit model of social dynamics underlying the observed trust relations,

$$X_t(n,m) \quad \text{s.t.} \;\; n,m = 1,\ldots,N = 18 \;\; \text{and} \;\; t = 1,2,3.$$

To this end we combine the mixed membership stochastic blockmodel for the observed relations (Airoldi et al., 2007) with a simple state-space model for the evolution of the latent aspects. The model is defined by the following three sets of equations,

$$\begin{aligned} P\left(\,\vec{\pi}_0(n) \mid \Theta\,\right) &\sim \mathbf{f}\circ \mathit{Gaussian}\left(\,\vec{0}, A\,\right), && (1)\\ P\left(\,\vec{\pi}_t(n) \mid \vec{\pi}_{t-1}(n), \Theta\,\right) &\sim \mathbf{f}\circ\left[\,\mathit{Gaussian}\left(\,\vec{0}, A\,\right) + \mathbf{f}^{-1}\circ\vec{\pi}_{t-1}(n)\,\right], && (2)\\ P\left(\,X_t(n,m) \mid \Pi_t, \Theta\,\right) &\sim \text{Bernoulli}\left(\,\vec{\pi}_t(n)'B\,\vec{\pi}_t(m)\,\right), && (3) \end{aligned}$$

for $n,m = 1,\ldots,N$ and $t = 1,2,3$; where $\mathbf{f}\circ\vec{x}$ and $\mathbf{f}^{-1}\circ\vec{x}$ are the logit transformation and its inverse, respectively, from $\mathbb{R}^k$ into the k-dimensional simplex and vice-versa; $\Theta = (A,B)$ is a collection of constant

quantities—A, B are matrices of size $k \times k$—that need to be estimated. Equation 2 implies that changing one's affiliation dramatically takes time, for reasonable values of A, thus membership vectors at time t depend heavily on their previous value. Equation 3 instantiates a simple stochastic blockmodel where trust depends on novices' faction allegiance at a given epoch, $\vec{\pi}_t(n), \vec{\pi}_t(m)$, and on the average trust among such factions, B. (Equation 1 is a technical necessity.) Posterior means for $\vec{\pi}_t(n)$ for all (n, t) in this model can be readily obtained via a Markov chain Monte Carlo sampler.

For a particular parameterization of A the posterior on the mixed membership vectors $\vec{\pi}_t(n)$ can be (arguably) approximated by fitting the model in Airoldi et al. (2007) to the collection of trust relations at each epoch independently, given that the latent factions can be uniquely identified.

Analysis. A preliminary set of posteriors means for the mixed membership vectors $\vec{\pi}_t(n)$ was obtained by fitting such a model, with no temporal smoothing. The results are displayed in Figure 1.

Discussion. Previous analyses—most of them focused on Breiger's collation of the data—suggest the existence of a community structure within the monastery. This supports Sampson's anthropological observation about the existence of three tight factions, but not the one about the interstitial members, arguably a fourth group. Our previous analysis of Breiger's data reaches a similar conclusion, from a novel perspective centered on mixed membership (Airoldi et al., 2007). The four sets of monks identified by Sampson are colors-coded in Figure 1 as follows: young turks in black, outcasts in blue, loyal opposition in red, and waverers in green.

The temporal dimension of the paired measurements about trust carries substantive information that is otherwise lost during data collation. A dynamic analysis of the evolution of trust identifies some differences with respect to the static analyses in the literature; for instance, the waverers—the green dots in Figure 1—appear as being truly interstitial members of the congregation, with respect to the three main factions, by the third time the questionnaire is administered—see the rightmost panel in the Figure. The suggestive power of mixed membership analysis is enhanced by the longitudinal data setting.

Attention to the evolution of the novices' memberships may offer some insights into the dramatic events that led to the failure of the monastic community. The landscape of social interactions within the congregation is shaped by the existence of informal factions. Using our model, we were able to measure two other intriguing aspects of the dynamics of novices' mixed membership: (i) a tendency towards the polarization of the three main factions; and (ii) the emergence of an interstitial group of individuals. This quantitative characterization of non-observable aspects underlying the dynamics of social interactions in the small monastery provides the elements for a theory of failure in isolated communities. The presence of informal groups weakens the structural cohesion of the congregation, the progressive polarization of such groups signals increasing differences, and the emergence of an interstitial group signals potential for conflict.

Conclusion. The proposed dynamic mixed membership analysis suggest a believable portrait of the events that led to the dissolution of the congregation in St. Mary's monastery. Inference results about the dynamics of novices' memberships substantiate Sampson's anthropological observations about the waverers, which previous attempts have not been able to support. We believe that our dynamic analysis of the evolution of trust among the novices may lead to the characterization of three basic elements of a theory of failure in isolated communities: weak social matrix, progressive polarization, and emergence of interstitial individuals.

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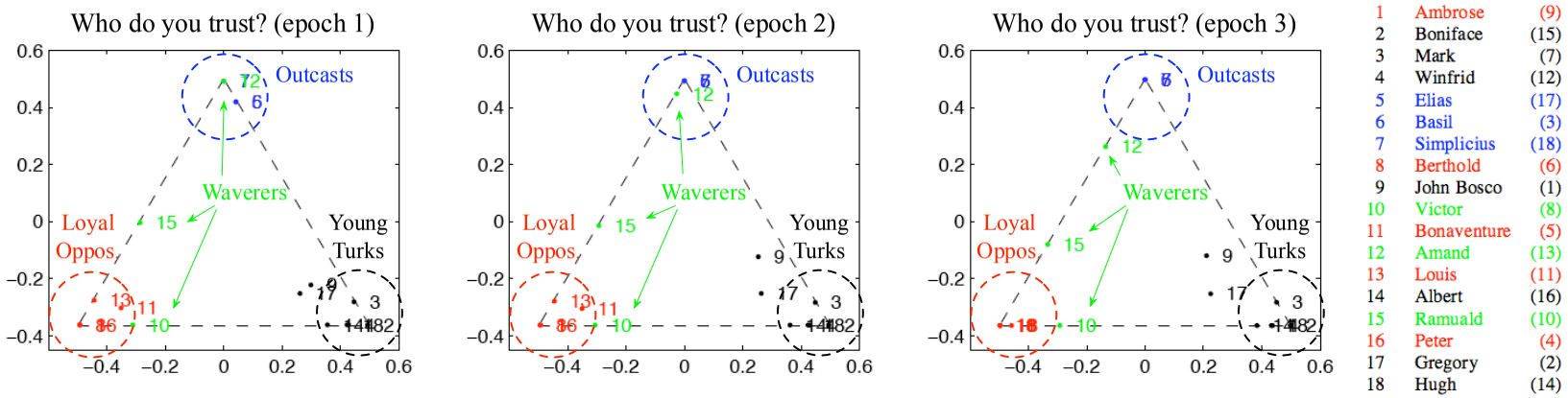

Figure 1: Posterior means for the mixed membership vectors, $\vec{\pi}_{1:18}$, projected in a 2-dimensional simplex. Numbered points can be mapped to monks' names using the legend on the right. The colors identify the four factions defined by Sampson's anthropological observations, as indicated by the colored text in each panel. The estimates correspond to a mixed membership stochastic blockmodel (Airoldi et al., 2007), with B set to the 3×3 identity matrix. (These results are preliminary; the model was applied to the trust relations at each epoch independently.)