Lecture 14

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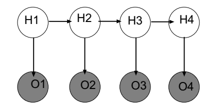
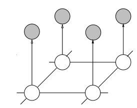
Generative vs discriminative models

- Generative model defines $P(h, o) = P(h)P(o|h)$.
  - HMM (hidden Markov model)
$$P(h, o) = [\prod_t P(h_t|h_{t-1})][\prod_t p(o_t|h_t)]$$
  - MRF (Markov random field)
$$P(h, o) = [\frac{1}{Z} \prod_i \prod_{j \in N_i} \psi_{ij}(h_i, h_j)] \prod_i P(o_i|h_i)$$

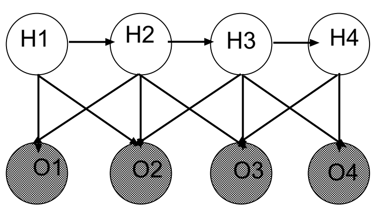


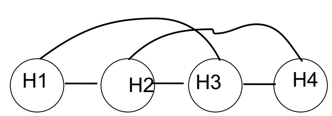


- Conditional model defines $P(h|o)$.
  - CRF (conditional random field)
$$P(h|o) = \frac{1}{Z(o)} \prod_i \prod_{j \in N_i} \psi_{ij}(h_i, h_j, o)$$

Advantages of CRFs

- Do not need to waste parameters modeling observed inputs o.
- Can use supervised machine learning methods to learn local evidence $P(h_i|o)$.
- Can incorporate arbitrary, nonlocal features of the input, without increasing complexity of inference.





Feature vectors for MRFs

- An MRF is
$$P(h) = \frac{1}{Z} \prod_c \psi_c(h_c)$$
- The clique potentials are often defined in terms of feature vectors
$$\psi_c(h_c) = \exp\left(\theta_c^T f_c(h_c)\right)$$
- By using indicator features, we can recover tabular potentials:
$$f_c(h_c) = [\delta(h_c, 1), \ldots, \delta(h_c, K)]$$

Feature vectors for CRFs

- A CRF is

$$P(h|v) = \frac{1}{Z(v)} \prod_c \psi_c(h_c, v)$$

- The (input-dependent) clique potentials are often defined in terms of feature vectors

$$\psi_c(h_c, v) = \exp\left(\theta_c^T f_c(h_c, v)\right)$$

- Example: logistic regression. Hidden node $H \in \{-1, +1\}$, cliques $=$ edges $= \{(H, v_c)\}$:

$$P(h|v) = \frac{1}{Z(v)} \prod_c e^{\theta_c v_c h}$$

- Example application: entity extraction from text (logistic regression with correlation amongst the hidden labels/ discriminative version of an HMM)

Entity Extraction

<u>Closed set</u>

U.S. states

He was born in <u>Alabama</u>…

The big <u>Wyoming</u> sky…

<u>Regular set</u>

U.S. phone numbers

Phone: <u>(413) 545-1323</u>

The CALD main office can be reached at <u>412-268-1299</u>

<u>Complex pattern</u>

U.S. postal addresses

University of Arkansas
<u>P.O. Box 140</u>
<u>Hope, AR 71802</u>

Headquarters:
<u>1128 Main Street, 4th Floor</u>
<u>Cincinnati, Ohio 45210</u>

<u>Ambiguous patterns, needing context and many sources of evidence</u>

Person names

…was among the six houses sold by <u>Hope Feldman</u> that year.

<u>Pawel Opalinski</u>, Software Engineer at WhizBang Labs.

Example: Features Used

Capitalized	Xxxxx
Mixed Caps	XxXxxx
All Caps	XXXXX
Initial Cap	X….
Contains Digit	xxx5
All lowercase	xxxx
Initial	X
Punctuation	.,:;!(), etc
Period	.
Comma	,
Apostrophe	‘
Dash	-
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- Character n-gram classifier says string is a person name (80% accurate)
- In stopword list (the, of, their, etc)
- In honorific list (Mr, Mrs, Dr, Sen, etc)
- In person suffix list (Jr, Sr, PhD, etc)
- In name particle list (de, la, van, der, etc)
- In Census lastname list; segmented by P(name)
- In Census firstname list; segmented by P(name)
- In locations lists (states, cities, countries)
- In company name list (“J. C. Penny”)
- In list of company suffixes (Inc, & Associates, Foundation)

- Hand-built FSM person-name extractor says yes, (prec/recall ~ 30/95)
- Conjunctions of all previous feature pairs, evaluated at the current time step.
- Conjunctions of all previous feature pairs, evaluated at current step and one step ahead.
- All previous features, evaluated two steps ahead.
- All previous features, evaluated one step behind.

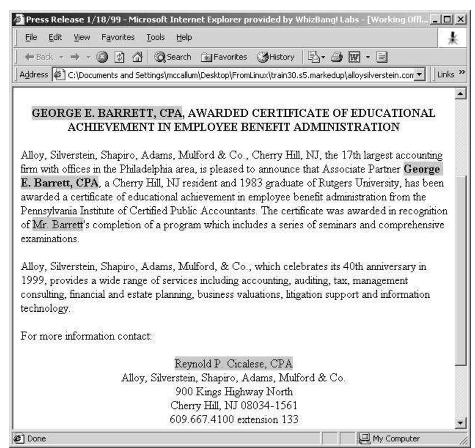
Total number of features = ~200k

Example: Person Name Extraction

Mallet Software

Person name Extraction

[McCallum 2001, unpublished]



Conditional models are not generative models of the data

- MaxEnt models are a generalized version of exponential family models, and so they can be thought of as generative models which assign probability distribution to joint settings of the features $f_i(\mathbf{x})$.
- But they *are not* generative models of the original inputs $\mathbf{x}$, because the features may be very complicated, nonlinear functions.
- Futhermore, it may be possible to generate joint feature settings which do not correspond to *any* possible input $\mathbf{x}$.
- For example, what if our generative model of English spelling gives $f_{\texttt{ing}}(c_1, c_2, c_3) = 1$ and $f_{?\texttt{ed}}(c_1, c_2, c_3) = 1$?

Parameter learning in MRFs/CRFs

- A CRF is $P(h|v) = \frac{1}{Z(v)} \prod_c \psi_c(h_c, v)$
- The (input-dependent) clique potentials are often defined in terms of feature vectors $\psi_c(h_c, v) = \exp\left(\theta_c^T f_c(h_c, v)\right)$
- Assume fully labeled data, (h^m, v^m) pairs. Log-likelihood is
$$\ell = \sum_m \log P(h^m|v^m) = \sum_m \sum_c \theta_c^T f_c(h_c^m, v^m) - \log Z(v^m)$$
where $Z(v^m) = \sum_h \prod_c \psi_c(h_c, v^m)$.
- Derivative of log-likelihood is
$$\frac{\partial \ell}{\partial \theta_c} = \sum_m f_c(h_c^m, v^m) - \sum_{h_c} P(h_c|v^m) f_c(h_c, v^m)$$
$$= \text{counts} - \text{expected counts}$$

Derivative of log-partition function

$$\begin{aligned}
Z_{v,\theta} &= \sum_h \exp\left(\sum_c \theta_c^T f_c(h_c, v)\right) \\
\frac{\partial \log Z_{v,\theta}}{\partial \theta_c} &= \frac{1}{Z_{v,\theta}} \frac{\partial}{\partial \theta_c} \sum_h \exp\left(\sum_c \theta_c^T f_c(h_c, v)\right) \\
&= \frac{1}{Z_{v,\theta}} \sum_h \frac{\partial}{\partial \theta_c} \exp\left(\sum_c \theta_c^T f_c(h_c, v)\right) \\
&= \frac{1}{Z_{v,\theta}} \sum_h \exp\left(\sum_c \theta_c^T f_c(h_c, v)\right) f_c(h_c, v) \\
&= \sum_{h_{c'}} \sum_{h_c} P(h_{c'}, h_c | v, \theta) f_c(h_c, v) \\
&= E_{h_c} f_c(h_c, v)
\end{aligned}$$

Parameter tieing

- Frequently we assumed tied parameters, to handle models of variable-size e.g., sequences of varying length, images of different size, web-data with multiple web-pages
$$\psi_c(h_c, v) = \exp(\theta^T f_c(h_c, v))$$
- In this case, we just sum the (expected) features over all cliques that share the same weight
$$\frac{\partial \ell}{\partial \theta_c} = \sum_m \left[\sum_c f_c(h_c^m, v^m)\right] - \left[\sum_c \sum_{h_c} P(h_c|v^m) f_c(h_c, v^m)\right]$$
- We can associate a weight with each type (class) of clique, to specify the tying pattern. This is called a *relational Markov network*.

Regularization

- We usually put a $\mathcal{N}(\theta; 0, \sigma^2 I)$ prior on the weights to do "soft" feature selection.
- The penalized log-likelihood is

$$\ell = \sum_m \sum_c \theta_c^T f_c(h_c^m, v^m) - \log Z(v^m) - \frac{\theta^T \theta}{2\sigma^2} + C$$

- Derivative of penalized log-likelihood is

$$\frac{\partial \ell}{\partial \theta_c} = \sum_m f_c(h_c^m, v^m) - \sum_{h_c} P(h_c|v^m) f_c(h_c, v^m) - \frac{\theta}{\sigma^2}$$

- The prior variance σ^2 is usually set by cross-validation.

Gradient ascent on log-likelihood

- Derivative of penalized log-likelihood is

$$\frac{\partial \ell}{\partial \theta_c} = \sum_m f_c(h_c^m, v^m) - \sum_{h_c} P(h_c|v^m) f_c(h_c, v^m) - \frac{\theta}{\sigma^2}$$

- This can be passed to any gradient-based optimizer, e.g., conjugate gradient or BFGS. This will find the global optimum (since fully observed, convex problem).
- There is an alternative method called iterative scaling, but it is slower, more complex and less general.
- Learning requires computing $P(h_c|v^m)$ for every clique c, every training case m, and every iteration of the gradient algorithm, so it can be very slow.

Approximate solution to exact log-likelihood

- If inference is intractable, we can approximate $P(h_c|v^m)$.
- If we use loopy belief propagation, one can show (Wainwright, Jaakkola, Willsky, AISTATS 03) that for pairwise tabular MRFs, one possible local optimum is

$$\hat{\theta}_{s,j} = \log \tilde{P}(X_s = j), \quad \hat{\theta}_{st,jk} = \log \frac{\tilde{P}(X_s = j, X_t = k)}{\tilde{P}(X_s = j)\tilde{P}(X_t = k)}$$

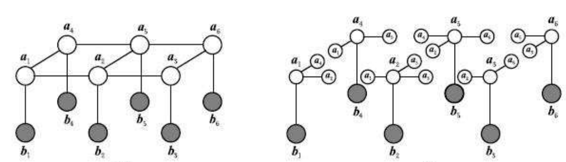
  so we can set the parameters from the empirical distribution $\tilde{P}$ without running BP.
- If we run BP with these parameters, one possible fixed point is that the model marginals will match the empirical marginals!
- However, this may not match the behavior of the true MLEs when the local evidence changes.
- For more general models, one can use approximate inference to compute an approximate gradient, but this may not converge.

Exact solution to approximate log-likelihood

- Instead of doing approximate inference, we can change the objective function:

$$\begin{aligned}
\ell(h^m|v^m) &= \frac{1}{Z(v^m)} \prod_i \phi_i(h_i^m, v^m) \prod_{j \in N_i} \psi_{ij}(h_i^m, h_j^m) \\
&\approx \prod_i \frac{1}{Z_i} \phi_i(h_i^m, v^m) \prod_{j \in N_i} \psi_{ij}(h_i^m, h_j^m) \\
&= \prod_i P(h_i^m | h_{N_i}^m, v^m)
\end{aligned}$$

where $Z_i = \sum_{h_i} \phi_i(h_i, v^m) \prod_{j \in N_i} \psi_{ij}(h_i, h_j^m)$



(a) (b)

Pseudo-likelihood approximation

- Pseudo-likelihood learns to trust its hidden neighbors too much (since they are assumed known during learning), hence leading to over-smooth estimates at run time.

$$\ell(h^m|v^m) \approx \prod_i P(h_i^m|h_{N_i}^m, v^m)$$

- One hack is to regularize the pairwise interaction potential ψ_{ij}.

Application: man-made building detection

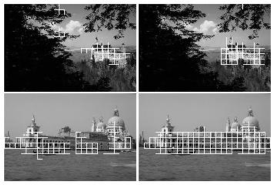
- "Discriminative Fields for Modeling Spatial Dependencies in Natural Images", Kumar & Herbert, NIPS 2003
- Goal: estimate $h_i \in \{-1, +1\}$ at each pixel i.
- Local-evidence defined in terms of features $f_i(v)$:

$$\phi_i(h_i, v; w) = \log \sigma(h_i w^T f_i(v))$$

- Image-dependent smoothing between neighboring labels

$$\psi_{ij}(h_i, h_j, v; \theta) = h_i h_h \theta^T g_{ij}(v)$$

- Inference= graph cuts, Learning= pseudo-likelihood



Feature induction

- We assumed $\psi_c(x_c) = \exp \theta_c^T f_c(x_c)$.
- Where do the features come from?
- McCallum (UAI 03) suggested a greedy feature induction scheme for 1D CRFs applied to text:
  - At each iteration, consider (in parallel) adding new atomic features (binary tests on the input) and conjunctions of existing features.
  - Evaluate quality of proposed candidates using the change in pseudo-likelihood.
  - Having chosen a set of features, add them and refit the weights using BFGS.
  - It learned features like $f_i(v) = \delta(v_i =' the', v_{i+1} =' of')$.
- Dietterich et al (ICML 04) suggested using boosting to solve a similar task.

CRFs with hidden variables

- Let v be visible, h be always hidden, and s be desired output state (observed in training).

$$\begin{aligned} P(s^m|v^m) &= \sum_h P(s^m, h|v^m) \\ &= \sum_h \frac{1}{Z(v^m)} e^{\Psi(s^m,h,v^m)} \\ &= \frac{\sum_h e^{\Psi(s^m,h,v^m)}}{\sum_h \sum_s e^{\Psi(s,h,v^m)}} \stackrel{\text{def}}{=} \frac{Z(s^m, v^m)}{Z(v^m)} \end{aligned}$$

where

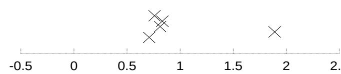
$$\Psi(s, h, v) = \sum_c \theta_c^T f_c(s_c, h_c, v)$$

Learning CRFs with hidden variables

- Log-likelihood
$$\log P(s^m|v^m) = \log Z(s^m, v^m) - \log Z(v^m)$$
- Derivative is
$$\frac{\partial \log P(s^m|v^m)}{\partial \theta_c} = E_h f_c(s^m, h, v^m) - E_h E_s f_c(s, h, v)$$
- Or we can use EM.
  - E-step: compute expected sufficient statistics.
  - M-step: maximize expected complete-data log-likelihood using standard techniques for fully observed MRFs (eg IPF or gradient).

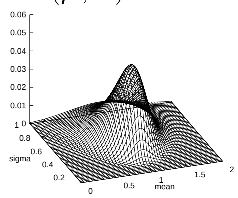
Hw5: MLEs for 1d Gaussians

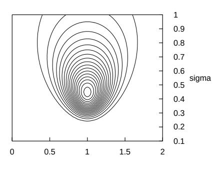
- Log-likelihood $\ell(\mu, \sigma) = \sum_n \log P(x_n|\mu, \sigma)$
- MLE for mean: $\hat{\mu}_{ML} = \frac{1}{N} \sum_n x_n$
- MLE for variance: $\hat{\sigma^2}_{ML} = \frac{1}{N} S$, where $S = \sum_n (x_n - \overline{x})^2$
- $\hat{\sigma^2}_{ML}$ is biased: $E_{X_{1:n} \sim \mathcal{N}(\mu,\sigma^2)} \hat{\sigma^2}_{ML}(X_{1:n}) = \frac{N-1}{N}\sigma^2$
- So we use $\hat{\sigma}^2_{N-1} = \frac{1}{N-1} S$
- Unbiased is not enough, e.g $E_{X_{1:n} \sim \mathcal{N}(\mu,\sigma^2)} \tilde{\mu}(X_{1:n}) = EX_1 = \mu$
- Also need consistency: $E(\hat{\theta} - \theta)^2 \to 0$.
  $\hat{\mu}$, $\hat{\sigma}^2_N$ and $\hat{\sigma}^2_{N-1}$ are all consistent.
- e.g., for data below, $N = 5$, $\overline{x} = 1.0$, $S = 1.0$, $\sigma_N = 1/\sqrt(5) = 0.45$, $\sigma_{N-1} = 1/\sqrt(4) = 0.5$.



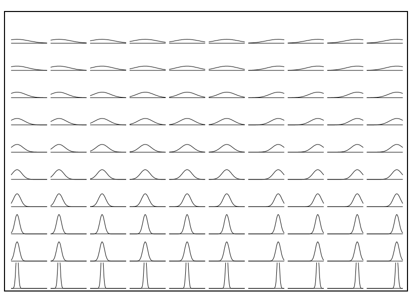
Bayesian updating in discrete hypothesis space

We plot likelihood $\ell(\mu, \sigma)$ for σ vs μ.

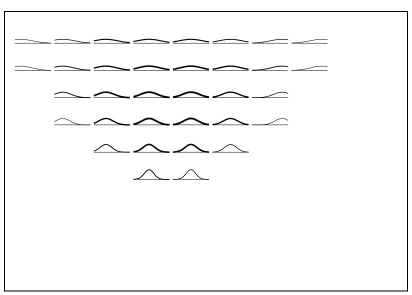




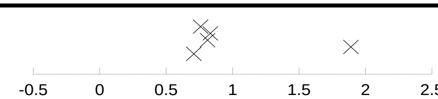
Prior



Posterior



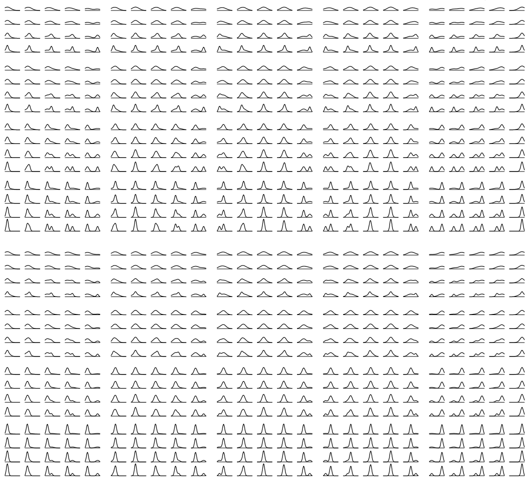
Mixture of 2 Gaussians



Maybe this data would be better fit by
$$p(x|\mu_1, \sigma_1, \pi_1, \mu_2, \sigma_2, \pi_2) =$$
$$= \frac{\pi_1}{\sqrt(2\pi)\sigma_1} \exp(-\frac{(x-\mu_1)^2}{2\sigma_1^2}) + \frac{\pi_2}{\sqrt(2\pi)\sigma_2} \exp(-\frac{(x-\mu_2)^2}{2\sigma_2^2})$$
where $\pi_1 + \pi_2 = 1$.

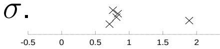
Prior hypothesis space for MOG2

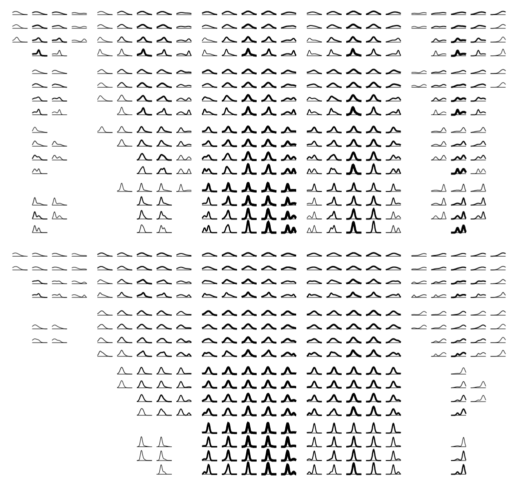
Top-half: $\pi_1 = 0.6$, bottom-half $\pi_1 = 0.8$. We plot μ vs σ.



Posterior hypothesis space for MOG2

Top-half: $\pi_1 = 0.6$, bottom-half $\pi_1 = 0.8$. We plot μ vs σ.





Model selection

- Maximum likelihood always picks the most complex model.
- Instead we should pick the most probable model $P(M|D) \propto P(D|M)$, where $P(D|M)$ is the *marginal* likelihood:

$$P(D|M) = \int_{\mu,\sigma} P(D|\mu,\sigma,M)P(\mu,\sigma|M)$$

- The integral over parameters penalizes overly complex models (Bayesian Occam's razor).
- This can be used for model selection (Bayesian version of hypothesis testing).
- Examples of model selection: number of clusters in K-means, order in K-th order Markov model, structure learning...