Announcements

What is new!

http://www.cs.ubc.ca/~mochetti/CPSC121.html
Goals

- Use truth tables to establish or refute the validity of a rule of inference.
- Determine whether or not a propositional logic proof is valid, and explain why it is valid or invalid.
- Explore the consequences of a set of propositional logic statements by application of equivalence and inference rules, especially in order to massage statements into a desired form.
- Devise and attempt multiple different, appropriate strategies for proving a propositional logic statement follows from a list of premises.
Argument

An argument is a sequence of statements ending in a conclusion. An argument is called valid if, and only if, whenever statements are substituted that make all the premises true, the conclusion is also true.

If $p$ then $q$

$p$

$\therefore q$

- An argument form is an abstract representation with variables.
- All statements in an argument are called premises (or assumptions or hypotheses).
- The final statement or statement form is called the conclusion.
- The symbol $\therefore$ means therefore and is placed before the conclusion.
An argument form is valid when no matter what particular statements are substituted for the statement variables in its premises, if the resulting premises are all true, then the conclusion is also true.

\[
\begin{align*}
p & \rightarrow q \lor \sim r \\
q & \rightarrow p \land r \\
\therefore & p \rightarrow r
\end{align*}
\]

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Rule of Inference

A rule of inference is a form of argument that is valid, such as modus ponens and modus tollens.

Modus Ponens

\[
p \rightarrow q \\
p \\
\therefore q
\]

Modus Tollens

\[
p \rightarrow q \\
\sim q \\
\therefore \sim p
\]
## Fallacy

A fallacy is an error in reasoning that results in an invalid argument, such as converse error and inverse error.

### Converse Error

| $p \rightarrow q$ | $q$ | $\therefore p$ |

### Inverse Error

| $p \rightarrow q$ | $\sim p$ | $\therefore \sim q$ |
Yay! Very well done overall!!! \o/

**Question 4:** Use the table below to determine whether this proposed rule of inference is valid. If the rule is invalid, select any one line of the truth table which proves that the rule is invalid.

\[
p \land \sim p \\
\therefore q
\]

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Proof

A rigorous formal argument that demonstrates the truth of a proposition, given the truth of the proof’s premises.

- A proof is used to convince other people (or yourself) of the truth of a conditional proposition.
- Every step must be well justified.
- Writing a proof is a bit like writing a function: you do it step by step, and make sure that you understand how each step relates to the previous steps.
Proofs and their Meaning

Things we might prove:

- We can build a combinational circuit matching any truth table.
- We can build any combinational logic circuit using only NOR gates.
- The maximum number of swaps we need to order \( n \) students is \( n \cdot (n - 1)/2 \).
- No general algorithm exists to sort \( n \) values using fewer than \( n \log_2 n \) comparisons.
- There are problems that no algorithm can solve.
Suppose that you proved this:

Premise 1

...

Premise \( n \)

\( \therefore \) Conclusion

Does it mean:

a. Premises 1 to \( n \) are true.

b. Conclusion is true.

c. Premises 1 to \( n \) are not a contradiction.

d. Conclusion isn’t a contradiction.

e. None of the above.
What does this argument mean?

Premise 1

...

Premise $n$

∴ Conclusion

a. Premise 1 $\land$ ... $\land$ Premise $n$ $\land$ Conclusion

b. Premise 1 $\lor$ ... $\lor$ Premise $n$ $\lor$ Conclusion

c. $(\text{Premise 1} \land ... \land \text{Premise } n) \rightarrow \text{Conclusion}$

d. $(\text{Premise 1} \land ... \land \text{Premise } n) \leftrightarrow \text{Conclusion}$

e. None of the above.
Let’s consider invalid the rule:

\[ p \rightarrow q \]

\[ q \]

\[ \therefore p \]

What can we say about the truth value of \( p \)?

- a. \( p \) is true
- b. \( p \) is false
- c. \( p \) might be either true or false
- d. \( p \) can be neither true nor false
A propositional logic proof is a sequence of propositions, where each proposition is one of either a premise or the result of applying a logical equivalence or a rule of inference to one or more earlier propositions.

- The last proposition is the conclusion.
- This is simpler than the more free-form proofs, only a limited number of choices at each step.
### Rules of Inference

#### Modus Ponens

$p \rightarrow q$

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#### Modus Tollens

$p \rightarrow q$

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### Generalization

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### Specialization

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## Rules of Inference

### Conjunction

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\[ p \rightarrow q \]

\[ q \rightarrow r \]

\[ \therefore p \rightarrow r \]
### Proof by Cases

Given:

- \( p \lor q \)
- \( p \rightarrow r \)
- \( q \rightarrow r \)

We want to show: \( r \)

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## Rules of Inference

### Resolution

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Prove that the following argument is valid:

\[
\begin{align*}
  p \lor q \\
  \sim p \\
  q \rightarrow \sim r \\
  r \lor (s \land t \land u) \\
  \therefore u
\end{align*}
\]
Example

Prove that the following argument is valid:

01. \( p \lor q \)
02. \( \sim p \)
03. \( q \rightarrow \sim r \)
04. \( r \lor (s \land t \land u) \)

\(\therefore\) \( u \)

Elimination from (01) and (02)
Modus Ponens from (03) and (05)
Elimination from (04) and (06)
Specialization from (07)
Prove that the following argument is valid:

01. \( p \lor q \)
02. \( \sim p \)
03. \( q \to \sim r \)
04. \( r \lor (s \land t \land u) \)
05. \( q \) \hspace{1cm} \textit{Elimination from (01) and (02)}
Prove that the following argument is valid:

01. \( p \lor q \)
02. \( \sim p \)
03. \( q \rightarrow \sim r \)
04. \( r \lor (s \land t \land u) \)
05. \( q \) \hspace{1cm} \text{Elimination from (01) and (02)}
06. \( \sim r \) \hspace{1cm} \text{Modus Ponens from (03) and (05)}
Example

Prove that the following argument is valid:

01. $p \lor q$
02. $\sim p$
03. $q \rightarrow \sim r$
04. $r \lor (s \land t \land u)$

05. $q$ \hspace{1cm} Elimination from (01) and (02)
06. $\sim r$ \hspace{1cm} Modus Ponens from (03) and (05)
07. $(s \land t \land u)$ \hspace{1cm} Elimination from (04) and (06)
Example

Prove that the following argument is valid:

01. \( p \lor q \)
02. \( \sim p \)
03. \( q \rightarrow \sim r \)
04. \( r \lor (s \land t \land u) \)
05. \( q \) \hspace{1cm} \text{Elimination from (01) and (02)}
06. \( \sim r \) \hspace{1cm} \text{Modus Ponens from (03) and (05)}
07. \( (s \land t \land u) \) \hspace{1cm} \text{Elimination from (04) and (06)}
08. \( u \) \hspace{1cm} \text{Specialization from (07)}

\[ \therefore u \]
Questions?

Ask CPSC 121

http://www.cs.ubc.ca/~mochetti/askCPSC121.html
The Fire-Trolls Problem

Fire-Trolls Problem\(^1\) from Quiz #4:

- **Premise 1:** If dragons are too scaly to portray dragons then trolls must be too smelly to play trolls, and vice versa.
- **Premise 2:** And yet, if the fire-trolls are correct, dragons are too scaly to portray dragons and yet trolls are not too smelly to play trolls.
- **Conclusion:** Therefore, the fire-trolls are incorrect, and dragons are not too scaly to portray dragons.

Note: fire-trolls are trolls portraying dragons in mystical theater.

\(^1\) taken from an article by Julian Baggini on logical fallacies.
Fire-trolls: which definitions should we use?

a. \( d = \) dragons, \( t = \) trolls, \( sc = \) scaly, \( sm = \) smelly, \( ft = \) fire-trolls, \( c = \) correct

b. \( d = \) dragons are too scaly, \( t = \) trolls are too smelly, \( pd = \) dragons portray dragons, \( pt = \) trolls portray trolls, \( o = \) fire-trolls are correct

c. \( d = \) dragons are too scaly to portray dragons, \( t = \) trolls are too smelly to portray trolls, \( o = \) fire-trolls are correct

d. None of these, but another set of definitions works well.

e. None of these, and this problem cannot be modeled well with propositional logic.
The Fire-Trolls Problem

Fire-trolls: do the two premises contradict each other (that is, is \( p1 \land p2 \equiv F \))?

a. Yes

b. No

c. Not enough information to tell
What can we prove?

- We **can** prove that the fire-trolls are wrong.
- We **can not** prove that dragons are not too scaly to play dragons.
- We **can not** prove that trolls are not too smelly to play trolls.
- We **can** prove that this argument is not valid.
The Fire-Trolls Problem

What can we prove?

- We **can** prove that the fire-trolls are wrong.
- We **can not** prove that dragons are not too scaly to portray dragons.
- We **can not** prove that trolls are not too smelly to play trolls.
- We **can** prove that this argument is not valid.

<table>
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<tr>
<th>d: dragons too scaly to play dragons</th>
<th>d</th>
<th>t</th>
<th>o</th>
<th>$d \leftrightarrow t$</th>
<th>$o \rightarrow d \land \sim t$</th>
<th>$\sim o \land \sim d$</th>
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</tr>
</tbody>
</table>

$$\therefore \sim o \land \sim d$$
Proof Strategies

- Look at the information you have:
  Is there irrelevant information you can ignore?
  Is there critical information you should focus on?

Work backwards from the end, especially if you have made some progress but are missing a step or two.

Don't be afraid of inferring new propositions, even if you are not quite sure whether or not they will help you get to the conclusion you want.

If you are not sure of the conclusion, alternate between:
- trying to find an example that shows the statement is false,
  using the place where your proof failed to help you design the counterexample.
- trying to prove it, using your failed counterexample to help you write the proof.
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Example: prove that the following argument is valid:

\[ p \\
p \rightarrow r \\
p \rightarrow \sim s \\
p \rightarrow (q \lor \sim r) \\
\sim q \lor s \\
\therefore s \]
Example: prove that the following argument is valid:

01. \( p \)
02. \( p \rightarrow r \)
03. \( p \rightarrow \neg s \)
04. \( p \rightarrow (q \lor \neg r) \)
05. \( \neg q \lor s \)

Modus Ponens from (01) and (02)
Modus Ponens from (01) and (04)
Elimination from (06) and (07)
Elimination from (05) and (08)
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02. \( p \rightarrow r \)
03. \( p \rightarrow \sim s \)
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05. \( \sim q \lor s \)

06. \( r \) \hspace{1cm} \text{Modus Ponens from (01) and (02)}
Example: prove that the following argument is valid:

01. \( p \)
02. \( p \rightarrow r \)
03. \( p \rightarrow \sim s \)
04. \( p \rightarrow (q \vee \sim r) \)
05. \( \sim q \vee s \)

06. \( r \)  \hspace{1cm} \text{Modus Ponens from (01) and (02)}
07. \( q \vee \sim r \)  \hspace{1cm} \text{Modus Ponens from (01) and (04)}
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04. \( p \rightarrow (q \lor \sim r) \)
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08. \( q \) \hspace{1cm} \text{Elimination from (06) and (07)}
09. \( s \) \hspace{1cm} \text{Elimination from (05) and (08)}
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01. \( p \)
02. \( p \rightarrow r \)
03. \( p \rightarrow \sim s \)
04. \( p \rightarrow (q \lor \sim r) \)
05. \( \sim q \lor s \)

06. \( \sim s \) \hspace{1cm} \text{Modus Ponens from (01) and (03)}
Example: prove that the following argument is valid:

01. $p$
02. $p \rightarrow r$
03. $p \rightarrow \sim s$
04. $p \rightarrow (q \lor \sim r)$
05. $\sim q \lor s$

06. $\sim s \quad \text{Modus Ponens from (01) and (03)}$

Since we can prove either $s$ and $\sim s$ this is a contradiction (there is no row with all premises true). Note that the argument is still valid, since there is not a case in which all premises are true.
Why can we not just use truth tables to prove propositional logic theorems?

a. No reason; truth tables are enough.
b. Truth tables scale poorly to large problems.
c. Rules of inference can prove theorems that cannot be proven with truth tables.
d. Truth tables require insight to use, while rules of inference can be applied mechanically.
Proof Strategies

Why not use logical equivalences to prove that the conclusions follow from the premises?

a. No reason; logical equivalences are enough.
b. Logical equivalences scale poorly to large problems.
c. Rules of inference can prove theorems that cannot be proven with logical equivalences.
d. Logical equivalences require insight to use, while rules of inference can be applied mechanically.
Questions?

Ask CPSC 121

http://www.cs.ubc.ca/~mochetti/askCPSC121.html
Prove that the following argument is valid:

\[ p \rightarrow q \]
\[ q \rightarrow (r \land s) \]
\[ \sim r \lor (\sim t \lor u) \]
\[ p \land t \]
\[ \therefore u \]
Prove that the following argument is valid:

01. \( p \rightarrow q \)
02. \( q \rightarrow (r \land s) \)
03. \( \sim r \lor (\sim t \lor u) \)
04. \( p \land t \)

05. \( p \) \hspace{1cm} Specialization from (04)
06. \( q \) \hspace{1cm} Modus Ponens from (01) and (05)
07. \( r \land s \) \hspace{1cm} Modus Ponens from (02) and (06)
08. \( r \) \hspace{1cm} Specialization from (07)
09. \( \sim t \lor u \) \hspace{1cm} Elimination from (03) and (08)
10. \( t \) \hspace{1cm} Specialization from (04)
11. \( u \) \hspace{1cm} Elimination from (09) and (10)
Given the following, what is everything you can prove?

\[ p \rightarrow q \]
\[ p \lor \sim q \lor r \]
\[ (r \land \sim p) \lor s \lor \sim p \]
\[ \sim r \]

Hercule Poirot has been asked by Lord Rumpd Dalton to find out who closed the lid of his piano after dumping the cat inside. Poirot interrogates two of the servants, Meece Pink and Jhyl Klone. One and only one of them put the cat in the piano. Plus, one always lies and one never lies.
- Jhyl Klone: I did not put the cat in the piano. Ayul Parn gave me less than $60 to help her study.
- Meece Pink: Jhyl Klone did it. Ayul Parn paid him $50 to help her study.

Who put the cat in the piano?
IF P IS FALSE, I WILL BE SAD.
I DO NOT WISH TO BE SAD.
 THEREFORE, P IS TRUE.

There. Now you can skip 99% of philosophical debates.