Level Set Methods: Dynamic Implicit Surfaces and the Hamilton-Jacobi Equation
CPSC 542D, Term 2, Winter 2006–2007

Course Details

Web Page: `http://www.cs.ubc.ca/~mitchell/Class/CS542D.2006W2/index.html`

Lectures: 14:00–15:30, Monday/Wednesday, ICICS/CS 238.

Instructor: Ian Mitchell, `mitchell@cs.ubc.ca`, ICICS 217.

Grading: Your grade will be based on 3–5 homework assignments, 1–2 exams, 1–2 in class presentations, and 1 project proposal. Relative weighting will depend on the precise number of each.

Textbook: *Level Set Methods and Dynamic Implicit Surfaces* by Stanley Osher & Ronald Fedkiw. Available at the bookstore.

Software: *A Toolbox of Level Set Methods* by Ian M. Mitchell (software and manual). Available from the course web page. Use version 1.1 beta.

Prerequisites: None officially. You should have been exposed in previous courses to elementary differential equations, multivariable calculus and introductory numerical methods. You do not need experience with partial differential equations (either analytic or numeric). Homeworks will be programmed in MATLAB, which is easy to pick up if you have experience in C, C++, Java or Fortran. There will be a tutorial on MATLAB outside of class in the first two weeks of term.

Breadth requirement: This course does not count toward the CS graduate course breadth requirement.

Project Proposal: Instead of a full project, you will write only a brief proposal describing a project that you might like to do which involves the material covered in this course. The final proposal (7–10 pages) will include motivation and related work as well as the steps necessary to complete the project, its goals and how you would evaluate whether those goals were met; however, you are **not** required to perform the research that you describe.

Homework collaboration policy: You may collaborate with other students in the class on homework questions prior to writing up the version that you will submit. This collaboration may include pseudo-code solutions to programming components. Once you begin writing the version that you will submit, you may no longer collaborate on that question, either by discussing the solution with other students, showing your solution to other students, or looking at the solutions written by other students. You may **never** share executable code (including MATLAB m-files) for homework questions. You may not re-submit an assignment solution from another course or a previous offering of this course without acknowledgement, regardless of authorship. You may not make your solution available as an aid to others.

You may seek help from the course instructor at any time while preparing your homework solutions. You may not receive help from any other person.

If you feel that you have broken this collaboration policy, you may specify in your homework solution the name of the person from whom you received help and what components of that help you feel were beyond the limits of this collaboration policy, in which case your grade will be suitably adjusted to take this collaboration into account. If you break this collaboration policy and fail to cite your collaborator, you will be charged with plagarism as outlined in the university calendar. If you have any questions or if you believe that other students have been breaking this policy, please contact the instructor.

Potential Topics: Not all will be covered. If you have specific interests from this list (or related topics not on this list), let the instructor know.

- Introduction: surfaces, sets, representations. Course details (prerequisites, assessment, text).
- Implicit surface functions, signed distance functions and constructive solid geometry. Computing geometric properties.
- Numerical approaches to differential equations: ODEs and time marching, finite differences, finite element, finite volume.
- Dynamic implicit surfaces (time-dependent Hamilton-Jacobi):
  - Motion by: convection, mean curvature, normal speed, combinations and constraints.
  - Computational details: boundary conditions, temporal and spatial derivatives, high order accuracy, timestep restrictions.
  - Reinitialization and construction of signed distance functions.
- Dynamic implicit surface applications:
  - Image restoration and segmentation.
  - Constructing surfaces from point clouds.
  - Free surface fluid flow.
- Advanced level set methods:
  - Narrow banding for efficient computation.
  - Particle level set method for accuracy.
  - Velocity extension for motion defined only on the front.
  - Vector level sets for curves in 3D.
  - Unstructured and adaptive computational meshes.
  - Implicit and semi-Lagrangian timestepping.
- The static Hamilton-Jacobi equation for first arrival:
  - Fast marching method for signed distance functions.
  - Dynamic programming.
  - Ordered upwind and sweeping methods for general equations.
- The time-dependent Hamilton-Jacobi:
  - Approximating general first-order terms.
  - Optimal control and zero-sum differential games.
  - Other terms: forcing, discounting, uncertainty.
  - Generating static HJ solutions.
- Theory: viscosity solution properties, other types of weak solution.
- Hamilton-Jacobi applications:
  - Option and derivative pricing.
  - Population dynamics.
  - Robotic path planning.
  - Reach sets for verification.