

# CPSC 322, Practice Exercise

## Solutions to Logic: Proofs

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### 1 Directed Questions

- Given a knowledge base  $KB$  and a conjunction of atoms  $g$ , what is meant by  $KB \models g$ ?  
**Answer:**  $g$  is a logical consequence of  $KB$ , that is, it is true in every model of  $KB$
- Given a proof procedure  $P$ , a knowledge base  $KB$  and a conjunction of atoms  $g$ , what is meant by  $KB \vdash_P g$ ?  
**Answer:**  $g$  can be derived from  $KB$  by means of the proof procedure  $P$ .
- Define what it means for a proof procedure to be sound.  
**Answer:**  $KB \vdash g$  implies that  $KB \models g$
- Define what it means for a proof procedure to be complete.  
**Answer:**  $KB \models g$  implies that  $KB \vdash g$
- What is the key idea of the bottom-up proof procedure?  
**Answer:** You search forward from the given knowledge base, iteratively expanding the set of atoms implied by the knowledge base  $KB$ .
- How do you know when you have completed a successful derivation using the bottom-up proof procedure?  
**Answer:** If the set of atoms implied by  $KB$  is a superset of the atoms in  $g$ , the derivation is successful.
- How can the bottom-up proof procedure show that there is no successful derivation?  
**Answer:** If the procedure finishes and the set of atoms implied by  $KB$  is not a superset of  $g$ , then there exists no successful derivation.
- What is the key idea of the top-down proof procedure?  
**Answer:** You search backward from the query to see if it can be derived from the knowledge base.
- How do you know when you have completed a successful derivation using the top-down proof procedure?  
**Answer:** You obtain the answer clause  $yes \leftarrow$  with an empty body.
- Give an example of an admissible heuristic for top-down search.  
**Answer:** The number of atoms in the clause, since it will take at least that many resolution steps, i.e. it won't overestimate.

## 2 Datalog

A university has asked you to write a program to help them determine whether or not to accept students who have applied for admission. There are 3 basic pathways for a student to be accepted. If a student is returning to the university after a time away and is in good academic standing with no outstanding fees, they are accepted. Students who submit a complete application and are qualified are also accepted. Students are qualified if they have high SAT scores as well as good high-school transcripts. The university also has a legacy program, wherein children of former graduates are qualified (though these student must still submit a complete application). For brevity, let's only talk about 3 individuals: Sam is a former graduate and Chris is his son. Chris has good high-school transcripts and he submitted a complete application. Laura is a returning student in good academic standing.

- Give the knowledge base representing this problem, using unary predicates `accepted`, `returning`, `goodStanding`, `clearBalance`, `appComplete`, `qualified`, `legacyStudent`, `highSAT`, `goodHS`, and `graduate`, as well as the binary predicate `child`. The university admissions officials should be able to provide queries such as `accepted(chris)` and get a true or false answer.

**Answer:** Here is a sample KB.

```
% file: school.pl
```

```
accepted(Student) <- returning(Student) & goodStanding(Student) & clearBalance(Student).
accepted(Student) <- appComplete(Student) & qualified(Student).
```

```
qualified(Student) <- legacyStudent(Student).
qualified(Student) <- highSAT(Student) & goodHS(Student).
```

```
legacyStudent(Student) <- child(Student, Parent) & graduate(Parent).
```

```
goodHS(chris).
graduate(sam).
child(chris, sam).
appComplete(chris).
```

```
returning(laura).
goodStanding(laura).
```

- Show the top-down derivation of the query `accepted(chris)` applied to your KB.

**Answer:**

```
yes <- [accepted(chris)].
yes <- [appComplete(chris), qualified(chris)].
yes <- [qualified(chris)].
yes <- [legacyStudent(chris)].
yes <- [child(chris, Parent), graduate(Parent)].
yes <- [graduate(sam)].
yes.
"yes" - proven
```

- Show one of the failing top-down derivations of the query *accepted(laura)* applied to your KB.

**Answer:**

yes  $\leftarrow$  [accepted(laura)].

yes  $\leftarrow$  [returning(laura), goodStanding(laura), clearBalance(laura)].

yes  $\leftarrow$  [goodStanding(laura), clearBalance(laura)].

yes  $\leftarrow$  [clearBalance(laura)].

no choice, fail

### 3 Learning Goals

You can:

- Define/read/write/trace/debug the BottomUp proof procedure
- Define/read/write/trace/debug the TopDown proof procedure
- Define/read/write/trace/debug the TopDown proof procedure as a search problem
- Represent simple domains in Datalog
- Apply TopDown proof procedure in Datalog