# University of Toronto at Scarborough <br> Department of Computer and Mathematical Sciences 

CSCD18: Computer Graphics
Midterm exam
Fall 2007
Duration: 50 minutes
No aids allowed
There are 6 pages total (including this page)

Family name:
Given names:
Student number:

| Question | Marks |
| :---: | :--- |
| 1 |  |
| 2 |  |
| 3 |  |
| 4 |  |
| 5 |  |
| Total | - |

1. [8 marks] Answer each of the following questions in one to three sentences:
(a) [2 marks] We have studied two very different classes of transformations: affine transformations and perspective projection. Basic properties of these two transformations can be summarized by considering what happens to lines (or points on those lines) as they are being transformed. In terms of lines, list one property common to both affine transformation and perspective projection; also list one property that hold for one but not the other transformation.
(b) [2 marks] Please describe, in words, what is pseudodeph and what it is used for?
(c) [1 marks] The standard pinhole camera does not contain a lens. Would the perspective projection formulation we derived for this camera in class change if we assume a presence of the thin lens at the pinhole? Why or why not?
(d) [3 marks] In real cameras the lens assembly is responsible for optical zoom. This allows photographers to zoom in on distant objects without physically getting closer to them. The ideal pinhole model discussed in class has no lens assembly; but we can still simulate zooming-in and out. In two or three sentences, explain how?
2. [8 marks] The diagram on the left shows a 2 D house object, with the origin denoted by the small circle. Define a transformation matrix $T$, in homogeneous coordinates, that transforms the house on the left to a larger and rotated house on the right. You may define $T$ as a composition of elementary transformation matrices.


3. [8 marks] Let $S$ be the 3D surface illustrated (called Möbius Strip after the German mathematicians August Ferdinand Möbius who discovered it in 1858). This surface can be defined in parametric form using the following equation:

$$
S(\alpha, \beta)=\left[\begin{array}{c}
2 \cos (\alpha)+\beta \cos (\alpha / 2) \\
2 \sin (\alpha)+\beta \cos (\alpha / 2) \\
\beta \sin (\alpha / 2)
\end{array}\right] \quad \begin{gathered}
0 \leq \alpha \leq 2 \pi \\
-0.4 \leq \beta \leq 0.4
\end{gathered}
$$

Find a vector that is normal to $S$ at point $\alpha=\pi, \beta=\frac{1}{10}$. Show and explain your work.

4. [5 marks] Assume we have defined a camera in terms of $\bar{e}, \vec{u}, \vec{v}$, and $\vec{w}$, where $\bar{e}$ denotes the eye location (the center of projection), and the vectors $\vec{u}, \vec{v}$ and $\vec{w}$ form a right-handed coordinate frame (i.e., $\vec{u}, \vec{v}$ and $\vec{w}$ provide the directions of the camera's $x, y$, and $z$ axes in the world coordinate frame). Let $\bar{p}^{c}$ be the representation of a point in camera-centered coordinates. Derive the homogeneous form of the transformation that maps the point $\bar{p}^{c}$ into its representation in world-centered coordinates, denoted $\bar{p}^{w}$.
5. [5 marks] Assume we have a pinhole camera with an origin at $\bar{e}$ and (virtual) image plane at a focal distance $f$. Derive the equation of a ray, in the camera-centered coordinate frame, that maps out all visible points in space that will be projected to a single point $\left(x^{*}, y^{*}\right)$ in the (virtual) image plane (assume far plane is at $\infty$ ).

