University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

CSCD18: Computer Graphics

Midterm Exam
Fall 2006

Duration: 50 minutes
No aids allowed

There are 6 pages total (including this page)

Family name: ________________________________

Given names: ________________________________

Student number: ________________________________

Question	Marks	
1	__________	/ 6 marks
2	__________	/ 8 marks
3	__________	/ 9 marks
4	__________	/ 3 marks
5	__________	/ 6 marks
Total	__________	/ 32 marks

1. A torus can be written in parametric form as $\bar{p}(\alpha, \beta) = (x(\alpha, \beta), y(\alpha, \beta), z(\alpha, \beta))$, where

$$
\begin{aligned}
x(\alpha, \beta) &= (R + r\cos\beta)\cos\alpha && (1)\\
y(\alpha, \beta) &= (R + r\cos\beta)\sin\alpha && (2)\\
z(\alpha, \beta) &= r\sin\beta && (3)
\end{aligned}
$$

r and R are constants and $\alpha, \beta \in [0, 2\pi)$.

(a) [3 marks] Derive the surface tangents at a point $\bar{p}(\alpha, \beta)$ as a function of α and β.

(b) [1 mark] What are the surface tangents at the point $\bar{p}(\frac{\pi}{2}, 0)$? (Reminder: $\cos(0) = 1, \sin(0) = 0, \cos(\frac{\pi}{2}) = 0, \sin(\frac{\pi}{2}) = 1$).

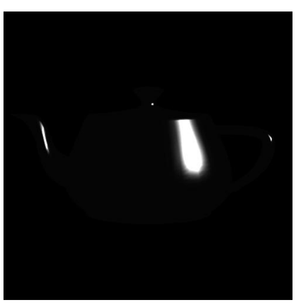
(c) [2 marks] Find the surface normal at the point $\bar{p}(\frac{\pi}{2}, 0)$?

2. (a) [5 marks] Give the mathematical form of the Phong reflectance model and give the names for the different model parameters.

(b) [3 marks] Each image below shows a teapot rendered with the Phong lighting model and a single light source (in OpenGL). Their only differences are due to the values of the surface reflection coefficients; for each image the coefficients are either zero or one. Specify, for each image below, the values of the Phong model reflection coefficients that are consistent with the appearance of the teapot (i.e., whether each is **zero** or **one**).

A:

B:



C:

D:

3. In the pinhole camera model, perspective projection models the geometry of the projection of light from a 3D surface point $\bar{p} = (p_X, p_Y, p_Z)$ to a location $\bar{q} = (x, y)$ on the viewplane. For this question, assume that we are given the 3D point $\bar{p}$ in camera-centered coordinates. As usual, we also assume that the optical axis is the negative $-Z$ axis, and that the viewplane is $Z = f$, for $f < 0$ (i.e., the viewplane is located at a distance $|f|$ in front of the pinhole).

   (a) [3 marks] State the mathematical equation for the perspective transform, i.e., specify $\bar{q} = (x, y)$ as a function of $\bar{p} = (p_X, p_Y, p_Z)$ and f.

   (b) [3 marks] Based on the geometry of projection (i.e., that light travels along straight lines), provide a derivation for the projection equation in (a).

(c) [3 marks] Re-express perspective projection in terms of homogeneous coordinates and the homogeneous transformation matrix, and explain why this is useful. Note: You do not need to worry about scaling the matrix so that the results lie within a canonical view volume. You do not need to worry about producing correct pseudodepth.

4. *[3 marks]* Why are BSP trees inappropriate for scenes in which all of the objects are moving?

5. *[6 marks]* Prove that, under affine transformations, 3D lines remain 3D lines. In doing so, specify how the direction of the line changes under an affine transformation.