University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

CSCD18: Computer Graphics

Midterm exam
Fall 2005

Duration: 50 minutes
No aids allowed

There are 4 pages total (including this page)

Answer in the spaces provided.
(If you need more space, you the backs of pages)

Family name: ______________________________

Given names: ______________________________

Student number: ______________________________

Question	Marks
1	__________
2	__________
3	__________
4	__________
5	__________
Total	__________

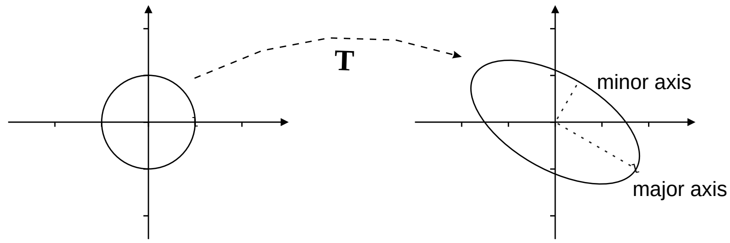
1. *[8 marks]* Let S be a 3D surface made up of points $\bar{p} = (x, y, z)$ that satisfy the implicit equation

$$4x^2 + 3y^2 + 2xz - 4 = 0 \ .$$

Find a vector that is normal to S at point $(1, 0, 1)$. Show and explain your work.

2. *[8 marks]* Assume we have defined a camera in terms of $\bar{e}$, $\vec{u}$, $\vec{v}$, and $\vec{w}$, where $\bar{e}$ denotes the eye location (the center of projection), and the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ form a right-handed coordinate frame (i.e., $\vec{u}$, $\vec{v}$ and $\vec{w}$ provide the directions of the camera's x, y, and z axes in the world coordinate frame). Let $\bar{p}^c$ be the representation of a point in camera-centered coordinates. Derive the homogeneous form of the transformation that maps the point $\bar{p}^c$ into its representation in world-centered coordinates, denoted $\bar{p}^w$.

3. *[8 marks]* Let C be a circle in 2D with radius one, centered at the origin as shown in the figure. Define a transformation matrix $\mathbf{T}$ that transforms the circle to an ellipse rotated clockwise by 30^o, with major axis of length 2 and minor axis length 1, as shown in the figure. (You may define $\mathbf{T}$ as a composition of elementary transformation matrices).



4. *[10 marks]* Suppose we define a 3D plane in parametric form as $\bar{p}(\alpha, \beta) = \bar{p}_0 + \alpha\vec{a} + \beta\vec{b}$. As usual, $\vec{a}$ and $\vec{b}$ are vectors, and $\bar{p}_0$ is a point. Further, let $\bar{r}(\lambda) = \bar{r}_0 + \lambda\vec{d}$ be a 3D ray with $\lambda \geq 0$, where $\bar{r}_0$ is a point, and $\vec{d}$ is a vector. Derive formulae to determine whether the ray intersects the plane, and, if it does, to compute the intersection point.

5. *[11 marks]* (a) In words, what is a *backface*?

(b) Explain (mathematically) how to perform backface culling for a triangle with vertices $\bar{p}_1$, $\bar{p}_2$ and $\bar{p}_3$ and outward-facing normal $\vec{n}$, where the eye of the camera is at location $\bar{e}$ with a gaze direction of $\vec{g}$.

(c) Sketch a simple diagram to show one example in which a triangle is not visible but would not be removed by backface culling.