University of Toronto
Department of Computer Science

CSCD18: Computer Graphics

Midterm exam
Fall 2004

Duration: 50 minutes
No aids allowed
Please write in pen

There are eight pages total (including this page)

Family name: ________________________________

Given names: ________________________________

Student number: ________________________________

Question	Marks
1	___________
2	___________
3	___________
4	___________
5	___________
6	___________
Total	___________

1. Answer each of the following in one to three sentences:

   [1 marks] (a) Back-face elimination (culling) does not completely remove all the surfaces of an object that are hidden from the camera. Sketch a figure to help justify the above claim.

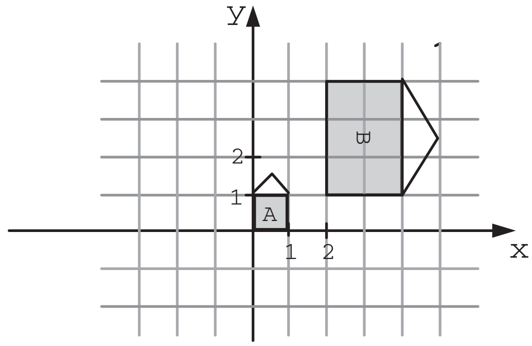
   [3 marks] (b) Before we can begin drawing with a z-buffer algorithm we need to initialize the values in the z-buffer. Why? What would be a reasonable value for initializing the z-buffer?

   [2 marks] (c) Why is there no lens in an ideal pinhole camera?

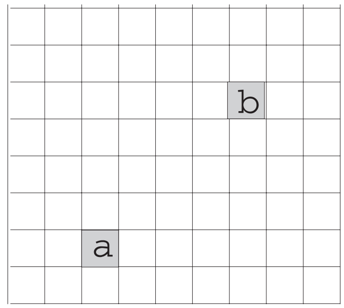
[3 marks] (d) In real cameras the lens assembly is responsible for optical zoom. This allows photographers to zoom in on distant objects without physically getting closer to them. The ideal pinhole model discussed in class has no lens assembly; but we can still simulate ”zooming-in and out”. In two or three sentences, explain how?

[3 marks] (e) Using a simple example explain why the *Painter’s Algorithm* may fail to produce an image with the correct occlusions depicted.

2. *[6 marks]* Find the 3×3 homogeneous matrix that transforms the 2D vertices of object $\mathbf{A}$ to the corresponding vertices of object $\mathbf{B}$ in the figure below. Express the matrix as a composition of elementary transformations namely translation, scaling, rotation, shear, and reflection.

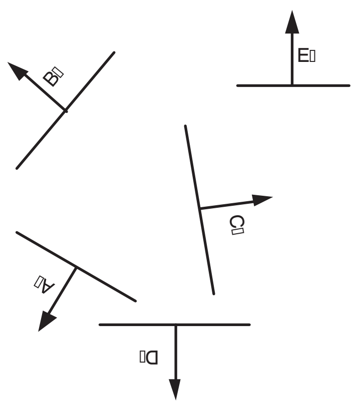


3. *[6 marks]* Given 2 points $\mathbf{a} = (a_x, a_y)$ and $\mathbf{b} = (b_x, b_y)$, where $b_y > a_y$ like those in the figure below, give pseudocode to rasterize the edge (as done in triangle rasterization). That is, for each scanline between $y = a_y$ and $y = b_y$, compute the pixel at which the line segment intersections the scanline.



4. *[10 marks]* (a) Figure 4 depicts a 3D scene of vertical surfaces, but viewed from above so all surfcaes appear as line segments. The line segments (walls) are labeled alphabetically, and the normals shown are outward facing (i.e., they indicate the front side of the walls (or line segments)). Build the BSP tree for this scene, inserting the line segments into the BSP tree in alphabetical order.

   Use the right child of a node to represent the +ve half-plane (or outward side) of its corresponding line segment. When dividing a line segment Z into two parts because Z is neither wholly outside (in front) or inside (behind) another line segment, say X, let Z_1 denote the part of Z that is +ve (or outward) side of X and let Z_2 denote the part of Z on the -ve (or inward) side of X.



5. *[4 marks]* (a) The parametric form of a 3D curve, parameterized by θ, is $(x(\theta), y(\theta), z(\theta))^T$. Give the 3D parametric equation of a circle in xy-plane with radius r and center $\mathbf{c} = (c_x, 0, 0)^T$.

[8 marks] (b) If we take that circle and rotate it about y-axis, the circle will effectively *sweep out* the surface of a torus (i.e., a donut). We can parameterize this rotation by ϕ and write the parametric equation of the torus as $(x(\theta, \phi), y(\theta, \phi), z(\theta, \phi))^T$. Derive parametric equations for $x(\theta, \phi)$, $y(\theta, \phi)$, and $z(\theta, \phi)$ in terms of θ, ϕ, r, and $\mathbf{c}$.

6. *[15 marks]* Derive the homogeneous perspective projection matrix that is used to project a point $\mathbf{p} = (p_x, p_y, p_z)^T$ through the eye, onto a plane $(\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{n} = 0$ (the equation of the plane in point-normal form). Here, $\mathbf{n}$ denotes plane normal, and $\mathbf{x}_0$ denotes a fixed point on the plane. Let the plane and the point be defined in viewer coordinates with the eye at the origin, and assume that point $\mathbf{p}$ does not lie on the plane.

   [Hint: Remember homogeneous perspective projection from class, where we let $\mathbf{M}$ be the homogeneous perspective matrix that projects a 3D point $\mathbf{p} = (p_x, p_y, p_z)^T$ along a line through the eye onto the near plane $z = n$, where $n < 0$ (i.e., the viewplane). So from $\widehat{\mathbf{p}} = (p_x, p_y, p_z, 1)^T$ we obtained $\widehat{\mathbf{q}} = (q_x, q_y, q_z, h)^T$ where $\widehat{\mathbf{q}} = \mathbf{M}\widehat{\mathbf{p}}$, and then after perspective division we obtained the projection of $\mathbf{p}$ onto the viewplane, i.e., $\mathbf{p}' = (\frac{q_x}{h}, \frac{q_y}{h}, n)^T$.]

END OF EXAM