UNIVERSITY OF TORONTO
Faculty of Arts and Science

APRIL / MAY 2005 EXAMINATIONS

CSC418H1S / CSC2504HS: Computer Graphics

Duration: 3 hours

No aids allowed

There are 11 pages total (including this page)

Given name(s): ________________________________

Family name: ________________________________

Student number: ________________________________

Question	Marks
1	___________/18
2	___________/ 8
3	___________/14
4	___________/10
5	___________/10
6	___________/17
7	___________/16
8	___________/10
9	___________/ 5
Total	__________/108

1 Shading

The Phong Illumination model can be summed up in the following equation:

$$E = r_a I_a + r_d I_d \max(0, \vec{n} \cdot \vec{s}) + r_s I_s \max(0, \vec{v} \cdot \vec{r})^{\alpha} \quad (1)$$

All vectors are unit length.

(a) [3 marks] The right hand side of the equation is the sum of three components. Give the name of each component (in left-to-right order).

(b) [4 marks] Draw a diagram illustrating the relationships of all the vectors, as well as the position of the eye and the light source.

(c) [3 marks] Basic (Whitted) ray tracing adds another summand to the equation. What is it? What illumination effect does it capture?

(d) [3 marks] Briefly explain how shadows are captured in basic ray tracing and how this may modify the equation above.

(e) [2 marks] What is a shadow effect that cannot be captured by the technique in (d)?

(f) [3 marks] Briefly describe a method that can produce more realistic shadows.

2 Curves

[8 marks] Derive a cubic polynomial $x(t)$ that satisfies the following constraints:

- $x(0) = 1$
- $x'(0) = 1$
- $x''(0) = 2$
- $x(1) = 5$

3 Projection

Suppose the viewplane is given by the equation $Ax + By + Cz - D = 0$, for points $\bar{x} = (x, y, z)^T$.

(a) [2 marks] Let $\bar{e} = (0, 0, 0)$ and $\bar{p} = (p_x, p_y, p_z)$. Give a parametric representation $\bar{q}(t)$ of the ray from $\bar{e}$ in the direction of $\bar{p}$ such that $\bar{q}(0) = \bar{e}$ and $\bar{q}(1) = \bar{p}$.

(b) [3 marks] Derive the value of t at the intersection between the ray and the viewplane.

(c) [2 marks] Derive the (Cartesian) coordinates of the intersection point.

(d) [2 marks] Give homogeneous coordinates for the intersection point that do not involve division.

(e) [3 marks] Write down a homogeneous matrix $\mathbf{M}$ such that $\mathbf{M}\hat{p}$ gives the homogeneous coordinates in (d). The same matrix should work for any point $\hat{p}$ in homogeneous coordinates (for which the intersection exists).

(f) [2 marks] If $A = B = 0$ and $C = 1$, what are $\mathbf{M}$ and the intersection point in homogeneous coordinates, written as a function of D?

4 Interpolative shading

Let $\bar{p}_0, \bar{p}_1$ and $\bar{p}_2$ be the vertices of a triangle in a triangular mesh and $\vec{n}_0, \vec{n}_1$ and $\vec{n}_2$ be the normals associated to $\bar{p}_0, \bar{p}_1$ and $\bar{p}_2$, respectively. For any point $\bar{p}$ and its associated normal $\vec{n}$, $C(\bar{p}, \vec{n})$ is the colour at the point $\bar{p}$ calculated using the Phong Illumination Model. Let $\bar{q}(\alpha, \beta) = (1 - \alpha - \beta)\bar{p}_0 + \alpha\bar{p}_1 + \beta\bar{p}_2$ be a point in the triangle.

(a) [2 marks] What are the valid ranges of the values α, β, and $\alpha + \beta$ for $\bar{q}$ to lie in the triangle?

(b) [2 marks] If Gouraud shading is applied, what is the colour at $\bar{q}$? (You may write this in terms of $C(\bar{p}, \vec{n})$; you do not need to explain how to evaluate $C(\bar{p}, \vec{n})$).

(c) [2 marks] If Phong shading is applied, what is the colour at $\bar{q}$? (You may write this in terms of $C(\bar{p}, \vec{n})$; you do not need to explain how to evaluate $C(\bar{p}, \vec{n})$).

(d) [4 marks] Explain briefly the difference between the two shading methods in terms of the accuracy and efficiency of the shading.

5 Surfaces and transformations

Let $f(\bar{p}) = 0$ be the implicit equation of a surface. An affine transformation Q maps $\bar{p}$ to $Q(\bar{p}) = A\bar{p} + \vec{t}$, where A is an invertible 3×3 matrix and $\vec{t}$ is a translation vector.

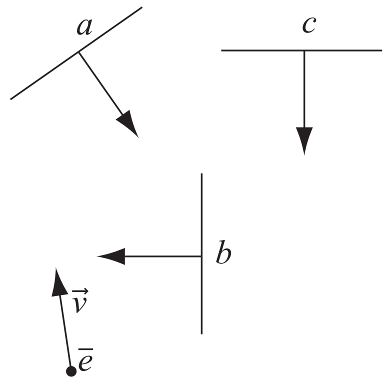
(a) [3 marks] Suppose we transform every point on the surface by Q. Give an implicit equation for this transformed surface.

(b) [2 marks] Express a normal $\vec{n}$ to the original surface at point $\bar{p}$ in terms of f and $\bar{p}$.

(c) [5 marks] Show that $((A^{-1})^T)\vec{n}$ is normal to the transformed surface at point $Q(\bar{p})$.

6 Visibility

The following diagram shows a cross-section of a scene, including faces and their outward normals. All faces are perpendicular to the plane containing the page.



(a) [3 marks] Show the BSP tree that would be constructed if the faces are processed in the order (a, b, c). Label each edge in the tree as "front" or "back." For this question, the "front" of a face is the side that the normal illustrated in the figure points to.

(b) [4 marks] Let $\bar{e}$ be the eye position. Describe the steps involved in traversing the tree and drawing the faces.

(c) [10 marks] Suppose we wish to cast a ray $\vec{r}$ from $\bar{e}$ in the direction $\vec{d}$. Give pseudocode for a recursive procedure that returns the first face that the ray hits, for any ray $\vec{r}$. The procedure should be efficient, i.e., it should not recur into any subtrees corresponding to partitions that the ray does not intersect. The procedure should be called as `intersectTree`$(f, \bar{e}, \vec{v})$, where f is the face at the root of the BSP tree, and $\bar{e} + \lambda\vec{v}$ is the ray to intersect. You may make use of the following helper functions (you do not need to write them yourself):

- `isLeaf`(f): return `true` if face f is a leaf of the BSP tree, and `false` otherwise.
- `nearSubtree`$(f, \bar{p})$: Given a point $\bar{p}$ and a face f, return the root of the subtree of f that contains $\bar{p}$ (or `null` if f is a leaf). The root of the subtree is represented as a face.
- `farSubtree`$(f, \bar{p})$: Given a point $\bar{p}$ and a face f, return the root of the subtree of f that does not contain $\bar{p}$ (or `null` if f is a leaf). The root of the subtree is represented as a face.
- `intersect`$(f, \bar{e}, \vec{v})$: Return `true` if the ray $\bar{e}+\lambda\vec{v}$ intersects the face f, and `false` otherwise.
- `intersectPlane`$(f, \bar{e}, \vec{v})$: Return `true` if the ray $\bar{e} + \lambda\vec{v}$ intersects the plane containing the face f, and `false` otherwise.

7 Curved surfaces

$\bar{p}_0, \bar{p}_1, \bar{p}_2$ and $\bar{p}_3$ are four points in 3D. A curve $\bar{c}(t)$ and a patch $\bar{b}(a,b)$ are defined as follows:

$$\bar{c}(t) = (1-t)^3\bar{p}_0 + 3t(1-t)^2\bar{p}_1 + 3(1-t)t^2\bar{p}_2 + t^3\bar{p}_3 \quad (2)$$
$$\bar{b}(a,b) = a(1-b)\bar{p}_0 + (1-a)(1-b)\bar{p}_1 + (1-a)b\bar{p}_2 + ab\bar{p}_3 \quad (3)$$

where $t \in [0,1], a \in [0,1], b \in [0,1]$. The points $\bar{p}_0$, $\bar{p}_1$, $\bar{p}_2$, and $\bar{p}_3$ may be distinct and non-coplanar.

(a) [2 marks] What type of curve is $\bar{c}(t)$? Give as specific a name as possible.

(b) [2 marks] What kind of patch is $\bar{b}(a,b)$? Give as specific a name as possible.

(c) [4 marks] Write down two intersection points of $\bar{c}(t)$ and $\bar{b}(a,b)$, and their corresponding t, a, and b values.

(d) [8 marks] Is $\bar{c}(t)$ always contained in $\bar{b}(a,b)$? If so, prove it. If not, derive the complete set of intersection points. *Hint:* Equate the coefficients of the parametric equations.

8 Radiance

Consider a surface point $\bar{p}$ with normal $\vec{n}$, illuminated by a single point light source with intensity I. The direction from $\bar{p}$ to the light source is $\vec{L}$. The BRDF of the surface is:

$$\rho(\vec{d}_i, \vec{d}_e) = r/\pi \tag{4}$$

where r is a constant material parameter. You may assume that all vectors are normalized.

(a) [4 marks] What is the irradiance from the light source at $\bar{p}$, in terms of I, $\vec{L}$, $\vec{n}$, and a differential solid angle $d\omega$?

(b) [4 marks] Derive the outgoing radiance from this surface point in direction $\vec{d}_e$. Simplify as much as possible. You may use the identity $\int_\Omega d\omega = \pi$, where $d\omega$ is a differential solid angle, and Ω is the hemisphere.

(c) [2 marks] What is a name for this type of surface? (There are two possible correct answers to this question).

9 Integration

[5 marks] Let $\vec{d}$ be a ray entering an image plane, and $I(\vec{d}, t)$ be the light entering the image along the ray at time t from an animated scene. If the camera shutter is open from t_0 to t_1, then the total light entering along this ray is:

$$\int_{t_0 \leq t < t_1} I(\vec{d}, t) dt \tag{5}$$

Give an algorithm for computing this light intensity by numerical integration. You may assume that a subroutine `rayTrace`$(\vec{d}, t)$ for computing $I(\vec{d}, t)$ is provided; you do not need to explain how to compute it.

END OF EXAM