University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

CSCD18: Computer Graphics

Final exam
Fall 2007

Duration: 3 hours
No aids allowed

There are 12 pages total (including this page)

Family name: ______________________________

Given names: ______________________________

Student number: ______________________________

Question	Marks
1	*[10 marks]* __________
2	*[13 marks]* __________
3	*[10 marks]* __________
4	*[17 marks]* __________
5	*[11 marks]* __________
6	*[17 marks]* __________
7	*[10 marks]* __________
8	*[12 marks]* __________
Total	__________

1. *[10 marks]* **Scanline Conversion.** Write an algorithm (it can be in pseudo-code form) for rasterizing the line segment defined by points $\bar{p}_1 = (p_1^x, p_1^y)$ and $\bar{p}_2 = (p_2^x, p_2^y)$. For simplicity, you can assume that $p_1^x < p_2^x$ and $p_1^y < p_2^y$. Also, find the color of each pixel rasterized by this algorithm, given that the color of $\bar{p}_1$ is 0.8 and $\bar{p}_2$ is 0.1.

2. *[13 marks]* **Lighting.** The Phong illumination model can be expressed using the following equation

$$E = r_a I_a + r_d I_d max(0, \vec{n} \cdot \vec{s}) + r_s I_s max(0, \vec{v} \cdot \vec{r})^{\alpha}$$

(a) *[3 marks]* In words, or in a picture, illustrate what are $\vec{s}$, $\vec{v}$ and $\vec{r}$ in the above stated Phong model correspond to.

(b) *[4 marks]* Explain why in both of the last two terms there is a max with a 0.

(c) *[2 marks]* What does α model? What would you expect α to be for an object with a mirror surface?

(d) *[2 marks]* In Whitted ray tracer we added an additional term to the above equation. What did that illumination term capture?

(e) *[2 marks]* How would you generalize the above equation to account for multiple light sources? Write down the resulting equation. What property of light does this generalization exploit?

3. *[10 marks]* **Shading.** In class we studied three shading models: Flat, Gouroud, and Phong.

   (a) *[3 marks]* List the three models in the order of increasing compute power required (*i.e.* with the fastest first).

   (b) *[3 marks]* What is the difference between Gouroud and Phong shading? Which one would you prefer for a specular object?

   (c) *[2 marks]* Assume that you want to render an object as precisely as possible but at the lowest possible computational cost. Which shading algorithm would you choose (out of the three listed above) for rendering a finely tessellated cube with a matte surface? Assume that the light is a point light source relatively close to the object.

   (d) *[2 marks]* Would your answer to the previous question change if we let the point light source be infinitely far away. Why?

4. *[17 marks]* **Transformations and Camera Projection.**

   (a) *[3 marks]* Describe the set of transformations that need to be applied to render a triangle defined in object-centric coordinate frame, within the rendering hierarchy.

   (b) *[4 marks]* Derive a world-to-camera transformation matrix for a camera that has an origin (eye point) at $\bar{o}$ (defined in global coordinate frame), a *look-at* vector $\vec{a}$, and a *top* vector, $\vec{t}$. For simplicity, you can assume that $\vec{a}$ and $\vec{t}$ are perpendicular.

   (c) *[10 marks]* Derive the transform matrix for scaling an object in 2D by a scale factor s along an arbitrary direction given by vector $\vec{u} = (u_x, u_y)$ rooted at $\bar{p} = (p_x, p_y)$. You can express your answer as a product of elementary 2D transformations.

5. *[11 marks]* **Surfaces.** Consider a 3D sphere that has an implicit equation of the following form:

$$(x - o_x)^2 + (y - o_y)^2 + (z - o_z) - r^2 = 0$$

where $\bar{o} = (o_x, o_y, o_z)$ is the center of the sphere and r is the radius.

(a) *[5 marks]* Compute the equation of the tangent plane to this sphere at an arbitrary point $\bar{p} = (p_x, p_y, p_z)$.

(b) *[4 marks]* Compute the equation of the normal of the sphere at the point $\bar{p} = (p_x, p_y, p_z)$.

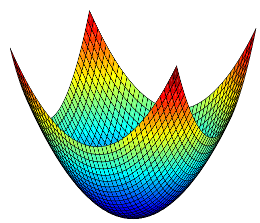
(c) *[2 marks]* Evaluate a normal of the unit sphere, $r = 1$, located at $\bar{o} = (5, 6, 11)$ at a point $\bar{p} = \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$.

6. *[17 marks]* **Ray Tracing.**

(a) *[10 marks]* Find an intersection of the ray, $\vec{r}(\lambda)$ with an infinite *elliptical paraboloid*, illustrated bellow, that can be expressed using the following equation:

$$z = \left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2$$

where a and b are real-valued constants and $-\infty < x < \infty$, $-\infty < y < \infty$. In terms of the intersection(s) describe which of the intersection points would correspond to a closest "hit point" that should be rendered.



(b) *[3 marks]* What is the difference between the basic (Whitted) ray tracer and Distribution Ray Tracing (DRT)? List at least 3 visual effects that can be achieved using DRT but not with Whitted ray tracer.

(c) *[2 marks]* What is BRDF and what is it used for?

(d) *[2 marks]* In a Whitted ray tracer, how do you know if a point on the surface is in shadow? Explain.

7. *[10 marks]* **Radiance.**

   (a) *[4 marks]* In words, define *radiance* and *irradiance*. Are both of these measured in the same units, if not, then what is the difference?

   (b) *[3 marks]* Imagine you have a sphere with a point light source (bulb) in the middle. Radiant intensity of the light source is $I = 10\ watts/sr$, what is the irradiance over the top half of the sphere. Does this quantity change as a function of the sphere radius?

   (c) *[3 marks]* If instead of the half-sphere, we consider an infinitesimally small patch would the irradiance on the patch change as the function of the sphere's radius? If so, then how?

8. *[12 marks]* **Interpolation and Animation.**

   (a) *[2 marks]* What is the difference between local and global control of splines? Why is the local control prefered for animation?

   (b) *[2 marks]* What is the difference between forward and inverse kinematics?

(c) *[8 marks]* Consider a 1D cubic interpolant, $x(t) = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3$, that goes through the following control points at the corresponding times t.

$$\begin{aligned} x(0) &= 2 \\ x(5) &= -5 \\ x(10) &= 1. \end{aligned}$$

This curve has also a constraint on the derivative, such that

$$x'(3) = 12.$$

Set up the equation for determining the coefficients $\alpha_0, \alpha_1, \alpha_2, \alpha_3$ of the basis functions for the cubic interpolant described above.