UNIVERSITY OF TORONTO AT SCARBOROUGH
Department of Computer and Mathematical Sciences

DECEMBER 2005 EXAMINATIONS

CSCD18H: Computer Graphics

Duration: 3 hours

Aids allowed: None

There are 15 pages total (including this page)

Please answer questions on the exam pages in the space provided. There are 7 main questions, most of which have multiple parts. Read questions carefully and answer as neatly, clearly and concisely as possible. Should a question be unclear or ambiguous, make a *reasonable* interpretation and state what you have assumed before answering.

Family name: ________________________________

Given names: ________________________________

Student number: ________________________________

Question	Marks
1	__________/ 14
2	__________/ 17
3	__________/ 9
4	__________/ 18
5	__________/ 8
6	__________/ 16
7	__________/ 18
Total	__________/ 100

1. Short Answer Questions [14 marks]

(a) [3 marks] What is subsurface scattering? Explain briefly why it cannot be represented as a BRDF.

(b) [2 marks] How would you compute a normal vector for a vertex of a 3D triangulated polygonal mesh, assuming that the mesh is intended to approximate a smooth surface?

(c) [2 marks] Give two advantages of key-frame animation over physics-based animation.

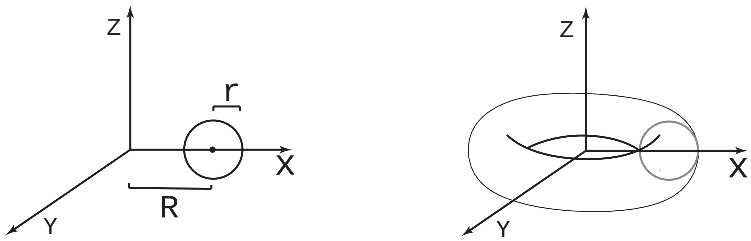
(d) [2 marks] Give two advantages of physics-based animation over keyframe animation.

(e) [3 marks] Describe the Painter's algorithm. Draw an example of a polygon or polygons that must be split to acheive correct visibility when rendering with the Painter's algorithm.

(f) [2 marks] In one or two sentences, and perhaps a simple diagram, explain what *squash and stretch* are used for in traditional animation.

2. Surface of Revolution [17 marks]

(a) [10 marks] A torus is surface of revolution. As shown in the figure, it can be generated by first defining a circle in the X-Z plane, and then revolving the circle about the Z axis. Derive a parametric equation of a torus that has major radius R and minor radius r, centred at the origin.



(b) [2 marks] Specify and explain any additional constraints that must exist for your parametric equation to model a torus. These include bounds on R, r, and any other parameters used.

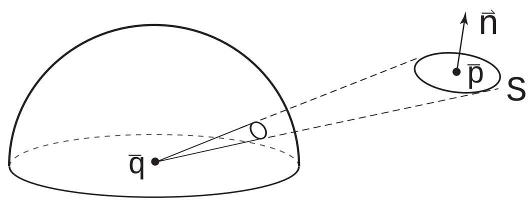
(c) [3 marks] Give a formula for the surface normal for the parametric form of the torus in part (a) above. You do not need to explicitly derive the derivatives.

(d) [2 marks] Given a point $\bar{p}_0$ on the torus and the normal $\vec{\mathbf{n}}_0$ at that point, found in part (c), give an implicit equation for the tangent plane at $\bar{p}_0$.

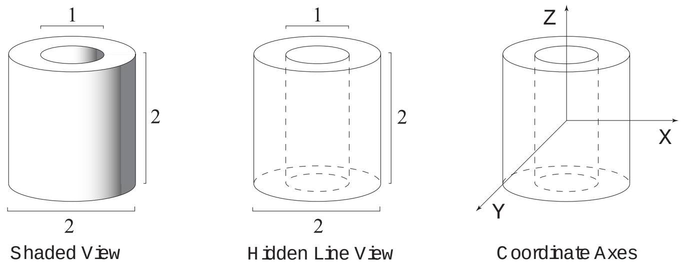
3. Solid Angle [9 marks]

(a) [2 marks] In words, what is solid angle?

(b) [7 makrs] As depicted in the figure below, let S be a small planar surface patch, centred at point $\bar{p}$ with unit normal $\vec{\mathbf{n}}$. Consider another point $\bar{q}$. Give a formula to compute the approximate size of S as seen from $\bar{q}$. That is, find an expression for the approximate solid angle subtended by S with respect to point $\bar{q}$. (*Hint:* The answer will depend on the area of S, the angle between the patch normal $\vec{\mathbf{n}}$ and the direction of $\bar{p}$ from $\bar{q}$, and on the distance from $\bar{p}$ to $\bar{q}$.)



4. Ray Intersections [18 marks] Your goal is to develop an algorithm to find intersections between an arbitrary ray $\vec{\mathbf{r}}(\lambda)$ and the shape shown in the figure. You need to find the intersection closest to the origin of the ray. We have broken down your task carefully into individual parts.



(a) [2 marks] Give an implicit-form equation for the wall of a right-circular cylinder, for which the circular cross-section has radius r_C, and the wall's height has length 2. As shown in the figure, let the cylinder be centred at the origin with its major axis along the Z-axis.

(b) [6 marks] Derive the equations for, and specify the steps of, a method for finding intersections between the ray $\vec{\mathbf{r}}(\lambda)$ and the wall of the cylinder in part (a). (Use back of page if necessary.)

(c) [4 marks] Derive the equations for, and specify the steps of, a method for finding intersections between a ray and a planar, circular disk, with radius r_D. Let the disk lie in a plane parallel to the X-Y plane, with its centre on the Z-axis with height z_D. You can assume that the origin of the ray does not lie in the plane containing the disk.

(d) [6 marks] Using the methods in Parts (b) and (c), give pseudocode to find the intersection between a ray and the object shown in the figure (comprising two cylindrical walls, the cap and base) that is closest to the origin of the ray.

5. Visibility [8 marks] For the following two scenarios, say whether Z-buffering or BSP-Trees would be a better choice of visibility (hidden surface removal) algorithm, and explain why.

(a) [4 marks] A large, static world filled with millions of polygons.

(b) [4 marks] A moderate-sized scene with hundreds of moving objects.

6. Curves and Surfaces [16 marks]

(a) [1 mark] Give an explicit equation for a cubic polynomial $x(t)$.

(b) [6 marks] Derive the cubic polynomial $x(t)$ that satisfies the following constraints:

$$x(0) = 2 \ , \quad x'(0) = 1 \ , \quad x''(0) = 2 \ , \quad x'''(0) = 6$$

That is, formulate and solve the system of equations to determine the polynomial coefficients.

(c) [3 marks] Bézier curves satisfy a *convex-hull property*. With a diagram and one or two sentences, explain the convex-hull property.

(d) [2 marks] In terms of parametric curves (such as Bézier curves), explain what *affine invariance* means?

(e) [4 marks] Show that Bézier curves (specifically, the Bernstein polynomial basis functions) are affine invariant.

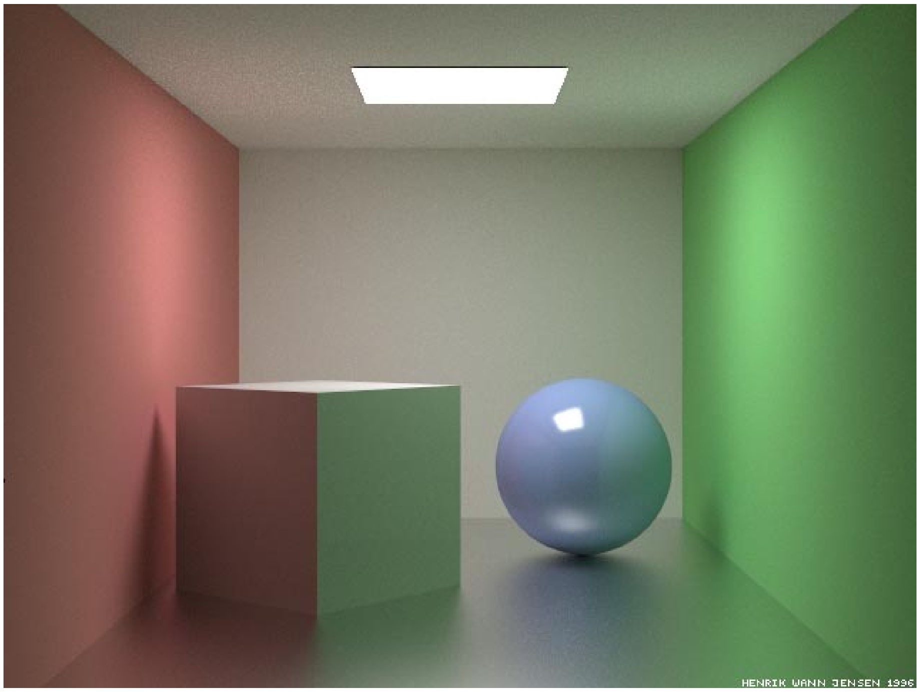
7. Illumination and Reflection [18 marks]

(a) [8 marks] Specify the mathematical form of the Phong reflectance model, given a surface point $\bar{p}$, its unit surface normal $\vec{\mathbf{n}}$, and the locations of the light source $\bar{l}$ and the pinhole of the camera $\bar{e}$. Define all other notational elements used in terms of $\bar{p}$, $\vec{\mathbf{n}}$, $\bar{l}$ and $\bar{e}$.

(b) [3 marks] For each of the three main terms of the Phong model, briefly describe what properties of visual appearance they can generate.

(c) [4 marks] Briefly explain the major ways in which basic (Whitted) ray tracing (with recursion) can generate properties of visual appearance beyond those possible with the Phong model.

(d) [4 marks] *Local illumination* only models the incident light at a surface that comes directly from a light source. For example OpenGL provides for local illumination. *Global illumination* describes all light that reaches the viewer, including light that bounces multiple times (secondary reflection) before reaching the viewer.



The image above shows a complex computer graphics scene, containing both local and global illumination effects. Name and describe two (2) global illumination effects that appear in the scene, with reference to specific regions in the image. You may circle and label specific regions for reference. Which of the effects might be possible with basic (Whitted) ray tracing?

END OF EXAM