UNIVERSITY OF TORONTO AT SCARBOROUGH

Department of Computer and Mathematical Sciences

DECEMBER 2004 EXAMINATIONS

CSCD18H: Computer Graphics

Duration: 3 hours

Aids allowed: None

There are 15 pages total (including this page)

Please answer questions on the exam pages in the space provided. Only use the backs of pages if necessary. There are seven main questions, most of which have multiple parts. Read questions carefully and answer as neatly, clearly and concisely as possible; you will be marked on both the clarity and correctness of your answers. Should a question be unclear or ambiguous, make a *reasonable* interpretation and state what you have assumed before answering.

Family name: ________________________________

Given names: ________________________________

Student number: ________________________________

Question	Marks
1	__________/12
2	__________/20
3	__________/ 6
4	__________/25
5	__________/ 8
6	__________/21
7	__________/ 8
Total	_________/100

1. Camera Models [12 marks]

(a) (2 marks) Under perspective projection, do parallel lines in 3D map to parallel lines in the 2D viewplane in general? Is parallelism preserved in special cases?

(b) (3 marks) Using a diagram and two or three sentences, explain how depth of field depends on the distance between the image plane and a thin lens (and aperture).

(c) (7 marks) If the device has 512×512 pixels, with the $(0, 0)$ point in the upper left corner, what is the transformation that maps points on the viewplane to points in device coordinates (in pixels)?

2. Illumination and Reflectance [20 marks]

(a) (10 marks) Using the Phong reflectance model, give the mathematical expression for the reflectance toward the eye $\vec{\mathbf{e}}$ (the pinhole) from a surface point $\vec{\mathbf{p}}$ with unit normal $\vec{\mathbf{n}}$, given a point light source at location $\vec{\mathbf{l}}$. Define any other variables needed by the model, and express all directions in terms of $\vec{\mathbf{p}}$, $\vec{\mathbf{n}}$, $\vec{\mathbf{e}}$, and $\vec{\mathbf{l}}$.

(b) (5 marks) In three or four sentences explain what are local and global models of illumination/reflectance (refer to natural phenomena, such as highlights, shadows, caustics, etc., and the ways that light and surfaces interact).

(c) (5 marks) In basic (Whitted) ray-tracing, rays are cast to estimate the incident illumination in directions of perfect specular reflection and refraction only. This is a limitation of basic ray-tracing, and helps to explain the characteristic look of many ray-traced images – sharp, specularly reflecting and/or transparent spheres. Explain why diffuse reflection and off-axis specular reflection are difficult to handle globally.

3. Shading [6 marks]

We introduced 3 polygon-based shading schemes in class, namely, flat, Gouraud, and Phong shading. Each scheme uses the Phong reflecance model.

(a) (4 marks) To shade a triangle that covers 30 pixels in the image, how many times will the Phong reflectance model be evaluated under each of the three shading schemes? Briefly explain your answer.

(b) (2 marks) In one or two sentences explain under what circumstances the image will look the same, regardless of whether it is rendered using flat, Gouraud or Phong shading.

4. Ray Tracing [25 marks]

Suppose that we wish to render some conic 3D solids (make up of a circular planar base and a conic wall), all of which are affine deformations of the generic implicit function for the conic wall given by

$$x^2 + y^2 - \frac{(1-z)^2}{4} \; = \; 0 \;\; , \qquad \text{where } |z| \leq 1 \; . \tag{1}$$

For example, one deformed cone we wish to ray-trace is given by (in world coordinates):

$$(x-2)^2 + \left(\frac{y}{3}\right)^2 - \frac{(z+1)^2}{4} \; = \; 0 \; , \tag{2}$$

and let the ray through a pixel (in world coordinates) be $\vec{\mathbf{r}}(\lambda) = \vec{\mathbf{a}} + \lambda\vec{\mathbf{b}}$, parameterized by λ.

(a) (9 marks) Sketch a 3D version of the generic cone relative to an $xyz-$axis, and find the 3D affine transformation, $(\mathbf{A}, \, \vec{\mathbf{t}})$, that deforms the generic cone (1) to the real-world cone (2).

(b) (4 marks) Instead of intersecting the ray $\vec{\mathbf{r}}(\lambda)$ with the real-world cone (2), we can intersect a deformed ray $\vec{\mathbf{r}}'(\lambda) = \vec{\mathbf{a}}' + \lambda\vec{\mathbf{b}}'$ with the generic cone (1). Give the expression for $\vec{\mathbf{a}}'$ and $\vec{\mathbf{b}}'$ in terms of $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, $\mathbf{A}$ and $\vec{\mathbf{t}}$.

(c) (12 marks) Derive an intersection method for finding the intersection(s) between the ray $\vec{\mathbf{r}}'(\lambda)$ and the *generic* cone. In doing so, compute the intersection *hit times* λ and the associated hit points. You may assume that the point $\vec{\mathbf{a}}'$ is *not* inside the generic cone.

5. Advanced Rendering [8 marks]

(a) (5 marks) Illustrate with a diagram why extended light sources often produce "soft" shadows. Also, provide a short explanation of how you could use ray tracing to handle extended light sources to simulate soft shadows.

(c) (3 marks) With a figure and one or two sentences describe how you can perform anti-aliasing in ray tracing.

6. Parametric Curves [21 marks]

(a) (8 marks) Let $\vec{\mathbf{c}}(t)$ be a 2D cubic curve, with first derivative $\vec{\mathbf{c}}'(t) = d\vec{\mathbf{c}}(t)/dt$, such that

$$\vec{\mathbf{c}}(0) = \vec{\mathbf{p}}_0 \ , \quad \vec{\mathbf{c}}(1) = \vec{\mathbf{p}}_1 \ , \quad \vec{\mathbf{c}}'(1) = \vec{\tau}_1 \ , \quad \vec{\mathbf{c}}'(2) = \vec{\tau}_2$$

Formulate a system of equations with which you can to solve for the unknown coefficients of the cubic curve. Show your work. **HINT:** Your solution may take the form $\mathbf{P} = \mathbf{C}\,\mathbf{Q}$ where $\mathbf{P} = [\vec{\mathbf{p}}_0\,,\ \vec{\mathbf{p}}_1\,,\ \vec{\tau}_1\,,\ \vec{\tau}_2]^T$ is the 4×2 matrix of known points and tangents, $\mathbf{C}$ is a 4×4 matrix, and $\mathbf{Q}$ is a 4×2 matrix that contains the unknown coefficients. (In your answer you are not required to actually solve for the inverse of $\mathbf{C}$.)

(b) (4 marks) Suppose we define a 2D curve with elements

$$x(t) \;=\; \begin{cases} t & \text{for } 0 \le t \le 2 \\ 2t-2 & \text{for } t > 2 \end{cases} \qquad\qquad y(t) \;=\; \begin{cases} 9t - 13t^2 + 4t^3 & \text{for } 0 \le t \le 2 \\ \sin(2t-4) - 2 & \text{for } t > 2 \end{cases}$$

What is the degree of continuity of the curve? Explain your answer.

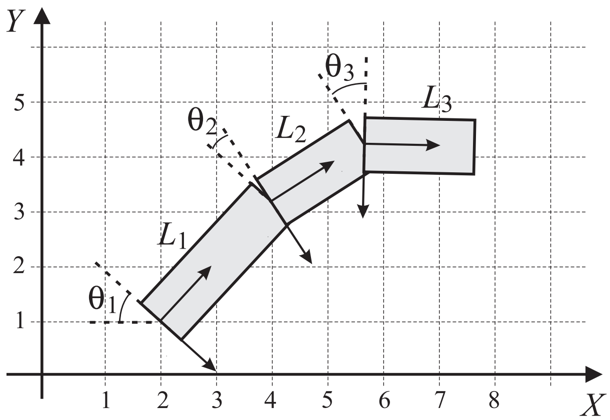
(c) (2 marks) Explain in two or three sentences what advantageous properties a cubic may have for modelling curves as compared to polynomial curves of degree greater than, or less than 3.

(d) (2 marks) Why are splines preferable to Bezier curves (or any single polynomial interpolant) when designing curves using many control points?

(e) (5 marks) Show how you would arrange 5 control points $\vec{\mathbf{p}}_j$ for $j = 1, 2, ...5$, in order to generate a degree 4 Bezier curve that is closed and everywhere C^1 continuous. Explicitly specify constraints on the placement of the control points, and the associated properties of Bezier curves that ensure C^1 continuity.

7. Hierarchical Models [8 marks]

Consider the 2-dimensional three link chain representing an *alien finger* above. Each link is attached to its parent by a one degree of freedom rotational joint (with axis normal to the page). When the finger is perfectly straight the rotation angles θ_2 and θ_3 are both zero. The lengths of the three parts, L_1, L_2 and L_3, are 3, 2 and 2 units respectively, and their local coordinate origins are centered at the base of each part. The finger is shown here in world coordinates XY.



(a) (4 marks) Given a point (a, b) on the finger tip, in the local coordinate frame of part L_3, write the sequence of transformation matrices that will transform this point to the world coordinate frame of reference.

(b) (4 marks) Suppose that L_1 and L_2 are held fixed with orientations given by angles θ_1 and θ_2. We now want to point the fingertip (i.e., L_3) in the direction of point (x, y) (given in world coordinates). Explain how to compute θ_3 so that L_3 points toward (x, y).

END OF EXAM