Announcements

- **Assignment 3**
  - **Programming** is now **out**
    - **Due** on Nov 14th (**next Friday**)
  - **Theory** (short) will be given out on **Nov 12th**
    - **Due** on **Nov 19th**

- **Assignment 3 Hints**
  - **Read the starter code carefully**
    - Only fill in the fragments of code that are needed (i.e. do not write your own classes/structures)
  - Make sure you **conceptually understand** what you need to do first
  - Ray tracing takes a while to render, so debugging can be slow (i.e. start right away)

Radiometry: Continuation

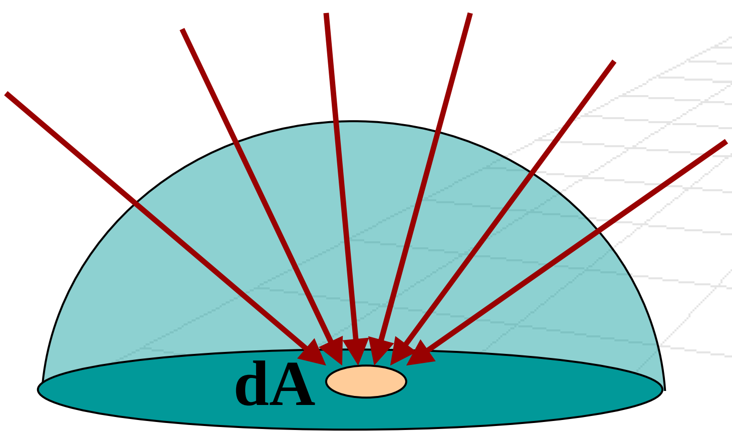
Computer Graphics, CSCD18

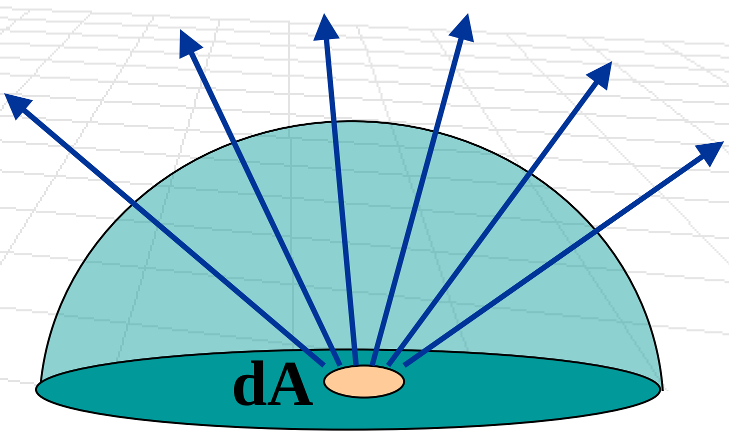
Fall 2008

Instructor: Leonid Sigal

Recall from last class…

- Light is manifested as photons
  - Number of photons at a point is zero
  - Hence, we going to talk about flux density (*i.e.* number of photons per unit area)
- **Irradiance** – amount of the light falling on the surface patch (measured in Watts/meters^2)
- **Radiance** – amount of light leaving the point per area (measured in Watts/(sr * meters^2))



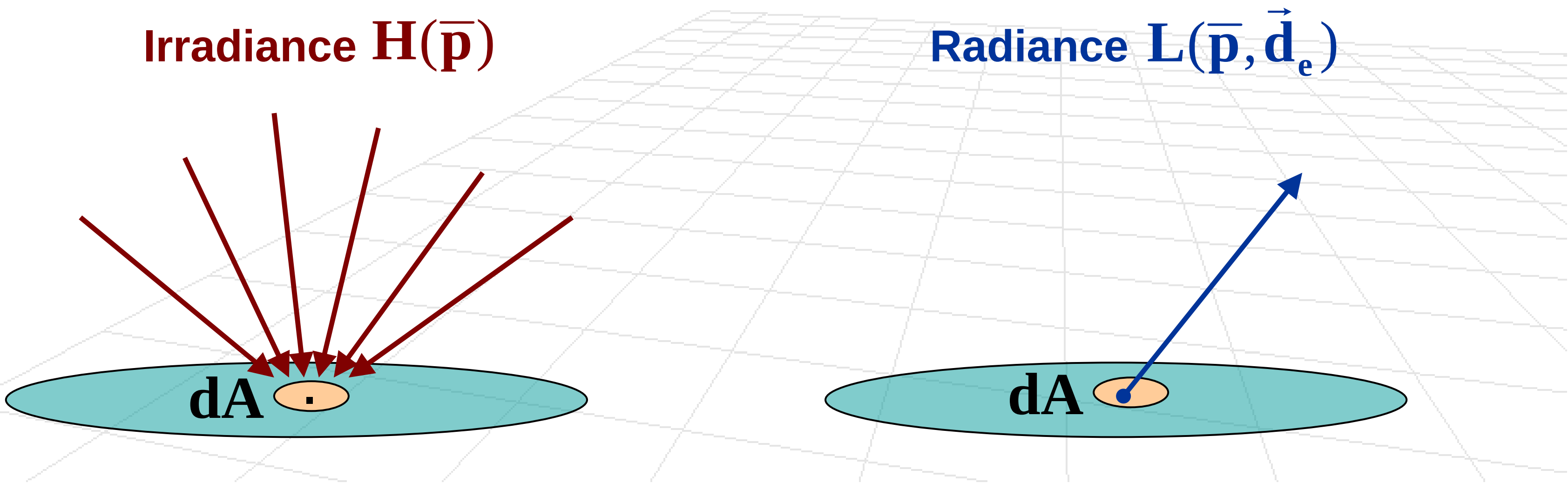


Bidirectional Reflectance Distribution Function (BRDF)

- **BRDF:** Ratio of emittant to incident light (i.e. radiance to irradiance)

$$\rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i) = \frac{\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e)}{\mathbf{H}(\overline{\mathbf{p}})}$$

Intuition: **what fraction of the light entering along one direction willbe emitted in the other**



Diffuse Reflection

- The only factor that determines appearance (radiance) of a **Lambertian** surface is irradiance (incident light)
- In other words, BRDF is constant and independent of incident and emittent direction. i.e. $\rho(\vec{\mathbf{d}}_{\mathbf{e}}, \vec{\mathbf{d}}_{\mathbf{i}}) = \rho_0$
- The radiance

$$\mathbf{L_d}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_{\mathbf{e}}) = \rho_0 \int_{\vec{\mathbf{d}}_{\mathbf{i}} \in \Omega_{\mathbf{i}}} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_{\mathbf{i}})(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_{\mathbf{i}}) \mathbf{d}\omega_{\mathbf{i}}$$

$$\mathbf{L_d}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_{\mathbf{e}}) = \rho_0 \int_{\vec{\mathbf{d}}_{\mathbf{i}} \in \Omega_{\mathbf{i}}} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_{\mathbf{i}}) \cos\theta_{\mathbf{i}} \mathbf{d}\omega_{\mathbf{i}}$$

- Since total irradiance must equal radiant exitance (conservation of energy), we can show that $\rho_0 = \frac{1}{\pi}$

Small proof

$$\int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_i) \cos\theta_i \mathbf{d}\omega_i = \int_{\vec{\mathbf{d}}_e \in \Omega_e} \rho_0 \int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_i) \cos\theta_i \mathbf{d}\omega_i \cos\theta_e \mathbf{d}\omega_e$$

$$\int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_i) \cos\theta_i \mathbf{d}\omega_i = \pi\rho_0 \int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_i) \cos\theta_i \mathbf{d}\omega_i$$

$$1 = \pi\rho_0$$

$$\rho_0 = \frac{1}{\pi}$$

Diffuse Reflection

- Despise simple BRDF, it's still hard to compute radiance because of the integral

$$\mathbf{L_d}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \int_{\vec{\mathbf{d}}_\mathbf{i} \in \Omega_\mathbf{i}} \mathbf{L}(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_\mathbf{i})(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i}) d\omega_\mathbf{i}$$

- Assuming point light source helps
  - Lets assume single point light source with intensity $\mathbf{I}$
  - Then irradiance is as before $\mathbf{H}(\bar{\mathbf{p}}) = \dfrac{\mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})}{\|\bar{\mathbf{p}} - \bar{\mathbf{e}}\|^2}$

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- Assuming that light is far away removes the denominator $\mathbf{L_d}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})$ **Why?**

Diffuse Reflection

- Despise simple BRDF, it's still hard to compute radiance because of the integral

$$\mathbf{L_d}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \int_{\vec{\mathbf{d}}_\mathbf{i} \in \Omega_\mathbf{i}} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_\mathbf{i})(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i}) \mathbf{d}\omega_\mathbf{i}$$

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Quantity is a constant for a surface

- Assuming that light is far away removes the denominator

$$\mathbf{L_d}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})$$

Why?

Remember the Phong model?

- Remember Phong lighting equation?

$$\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{c}}) = \mathbf{r_d I_d} \max(0, \vec{\mathbf{d}}_\mathbf{i} \cdot \vec{\mathbf{n}}) + \mathbf{r_a I_a} + \mathbf{r_s I_s} \max(0, \vec{\mathbf{r}} \cdot \vec{\mathbf{c}})^\alpha$$

$$\mathbf{r_d} = \rho_0 \overset{?}{\leq} \frac{1}{\pi}$$

- Assuming that light is far away removes the denominator

$$\mathbf{L_d}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})$$

Ambient Illumination

- **Remember:** we need ambient illumination, because diffuse lighting looks artificial (parts of the object are black)

- Ambient illumination is equivalent to uniform illumination and constant BRDF (as in the diffuse case)

$$\mathbf{L_a}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e) = \rho_a \int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i)\, \mathbf{d}\omega_i$$

- It's easy to see that the integral in the above equation is simply a constant

$$\mathbf{L_a}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e) = \rho_a \mathbf{I_a}$$

Remember the Phong model?

- Remember Phong lighting equation?

$$\mathbf{L}(\overline{\mathbf{p}},\vec{\mathbf{c}}) = \mathbf{r_d I_d}\max(0,\vec{\mathbf{d}}_\mathbf{i}\cdot\vec{\mathbf{n}}) + \boxed{\mathbf{r_a I_a}} + \mathbf{r_s I_s}\max(0,\vec{\mathbf{r}}\cdot\vec{\mathbf{c}})^\alpha$$

$$\mathbf{r_a} = \rho_\mathbf{a}$$

- It's easy to see that the integral in the above equation is simply a constant

$$\mathbf{L_a}(\overline{\mathbf{p}},\vec{\mathbf{d}}_\mathbf{e}) = \rho_\mathbf{a}\mathbf{I_a}$$

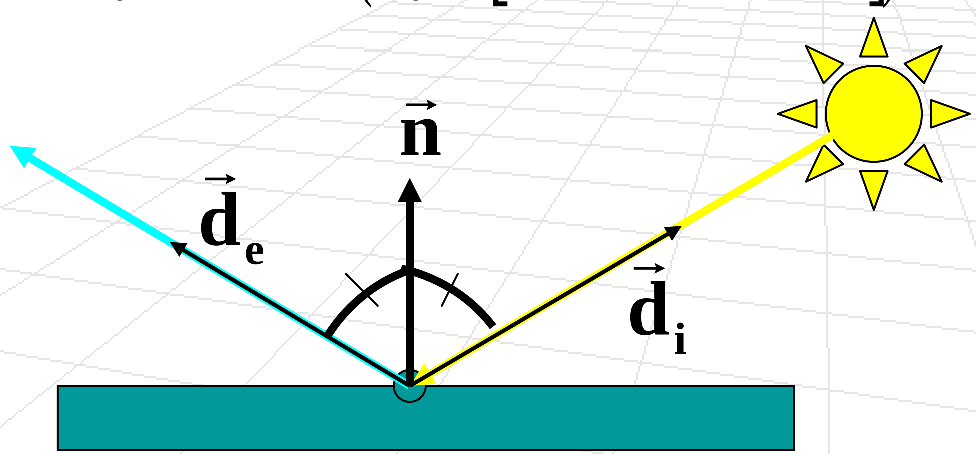
Specular Reflection

- For specular (mirror) surfaces each incident direction is reflected toward unique emittant direction
- The emittant direction can be derived as before in the Phong model

$$\vec{\mathbf{d}}_{\mathbf{e}} = 2(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_{\mathbf{i}})\vec{\mathbf{n}} - \vec{\mathbf{d}}_{\mathbf{i}}$$

- Since all of the light is reflected into a single direction, the corresponding BRDF can be formulated as follows:

$$\rho(\vec{\mathbf{d}}_{\mathbf{e}}, \vec{\mathbf{d}}_{\mathbf{i}}) \propto \delta\left(\vec{\mathbf{d}}_{\mathbf{e}} - \left[2(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_{\mathbf{i}})\vec{\mathbf{n}} - \vec{\mathbf{d}}_{\mathbf{i}}\right]\right)$$



Specular Reflection

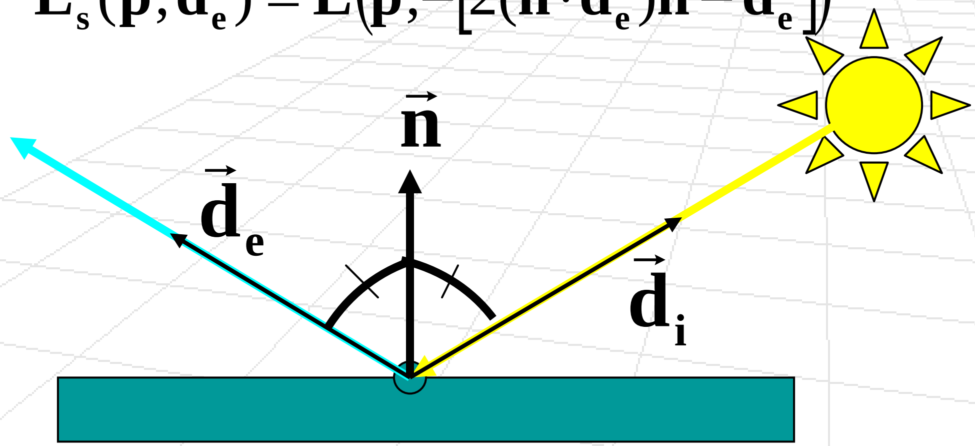
- If we assume that light emitted is the same amount of light incident (conservation of energy), we can derive the proportionality constant

$$\rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i) = \frac{1}{\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i} \delta\left(\vec{\mathbf{d}}_e - \left[2(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i)\vec{\mathbf{n}} - \vec{\mathbf{d}}_i\right]\right)$$

- Specular radiance can then be computed as for other components $\mathbf{L}_s(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e) = \int_{\vec{\mathbf{d}}_i \in \Omega_i} \rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i) \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i) \mathbf{d}\omega_i$

  which simplifies in this case to:

$$\mathbf{L}_s(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e) = \mathbf{L}\left(\overline{\mathbf{p}}, -\left[2(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_e)\vec{\mathbf{n}} - \vec{\mathbf{d}}_e\right]\right)$$



Off-axis Secularity

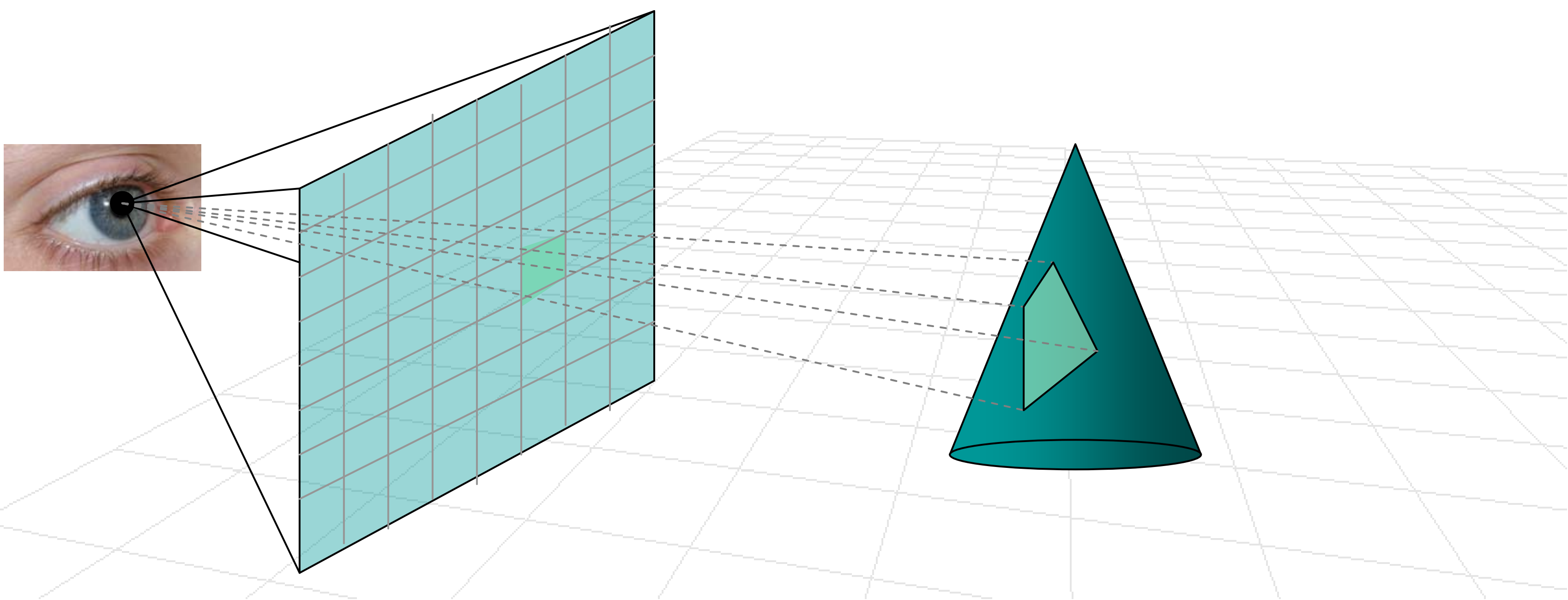
- If we have more complex surfaces (not just mirrors) we will have off-axis secularities
- In that case the BRDF will not be a simple delta function and we need to go back to the full integral formulation for the radiance
- Phong model makes the point light source assumption that is far away, this leads to the approximation we already encountered

$$\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{c}}) = \mathbf{r_d I_d} \max(0, \vec{\mathbf{d}}_\mathbf{i} \cdot \vec{\mathbf{n}}) + \mathbf{r_a I_a} + \boxed{\mathbf{r_s I_s} \max(0, \vec{\mathbf{r}} \cdot \vec{\mathbf{c}})^{\alpha}}$$

How will all of this help in Ray Tracing?

- We will consider a more accurate (and much more expensive) approximation to the radiance at the "hit point" based on the integral of the BRDF and incident irradiance

- What do we integrate over?
  - We integrate over area of a pixel



Distribution Ray Tracing

Computer Graphics, CSCD18

Fall 2007

Instructor: Leonid Sigal

Distribution Ray Tracing

- In **Whitted Ray Tracing** we computed lighting very crudely
  - Phong + specular global lighting
- In **Distributed Ray Tracing** we want to compute the lighting as accurately as possible
  - Use the formalism of Radiometry
  - **Compute irradiance at each pixel** (by integrating all the incoming light)
  - Since integrals are can not be done analytically, we will employ **numeric approximations**

Benefits of Distribution Ray Tracing

- Better global diffuse lighting
  - Color bleeding
  - Bouncing highlights
- Extended light sources
- Anti-aliasing
- Motion blur
- Depth of field
- Subsurface scattering

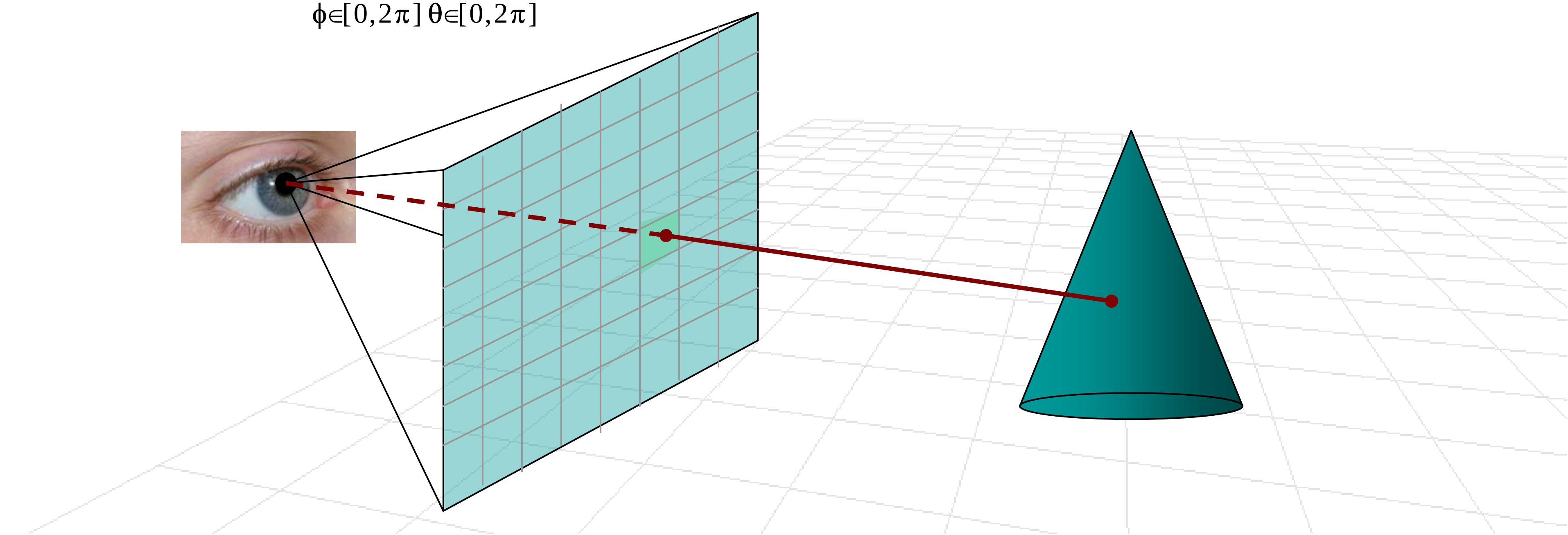
Radiance at a Point

- Recall that radiance (shading) at a surface point is given by

$$\mathbf{L}(\overline{\mathbf{p}},\vec{\mathbf{d}}_{\mathbf{e}}) = \int_{\Omega} \rho(\vec{\mathbf{d}}_{\mathbf{e}},\vec{\mathbf{d}}_{\mathbf{i}})\mathbf{L}(\overline{\mathbf{p}},-\vec{\mathbf{d}}_{\mathbf{i}})(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_{\mathbf{i}})\,\mathbf{d}\omega$$

- If we parameterize directions in spherical coordinates and assume small differential solid angle, we get

$$\mathbf{L}(\overline{\mathbf{p}},\vec{\mathbf{d}}_{\mathbf{e}}) = \int_{\phi\in[0,2\pi]}\int_{\theta\in[0,2\pi]} \rho\left(\vec{\mathbf{d}}_{\mathbf{e}},\vec{\mathbf{d}}_{\mathbf{i}}(\phi,\theta)\right)\mathbf{L}\left(\overline{\mathbf{p}},-\vec{\mathbf{d}}_{\mathbf{i}}(\phi,\theta)\right)\left(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_{\mathbf{i}}(\phi,\theta)\right)\sin\theta\,\mathbf{d}\theta\,\mathbf{d}\phi$$



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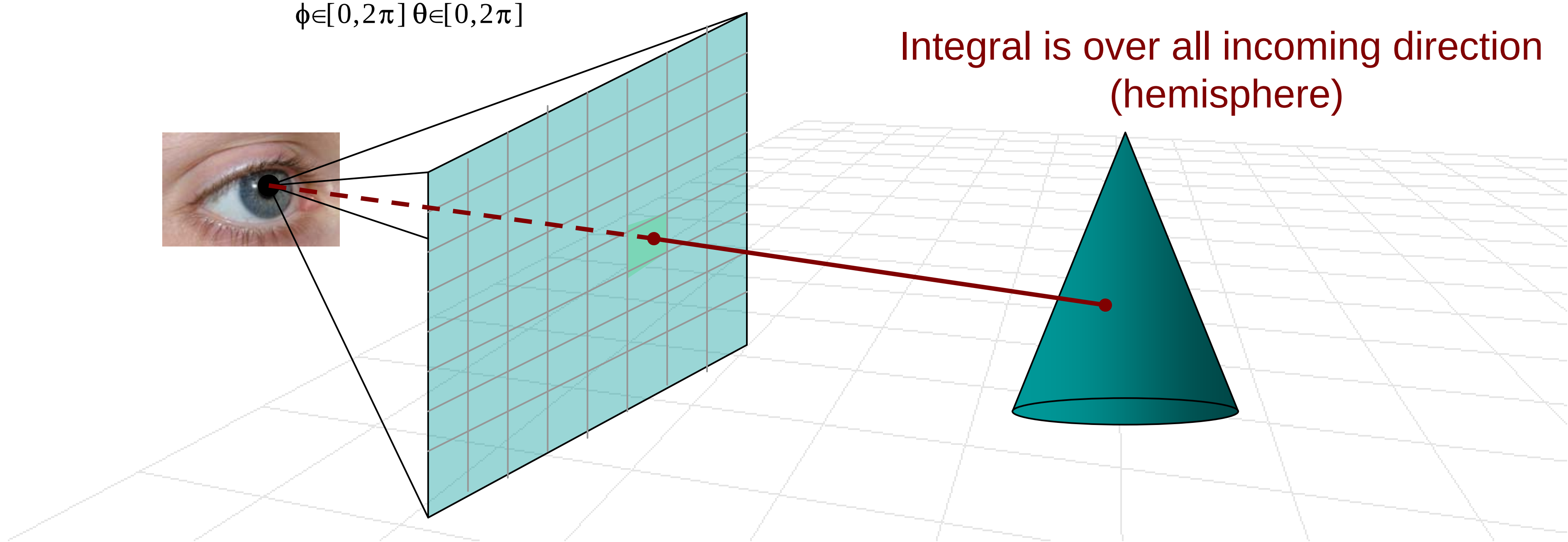
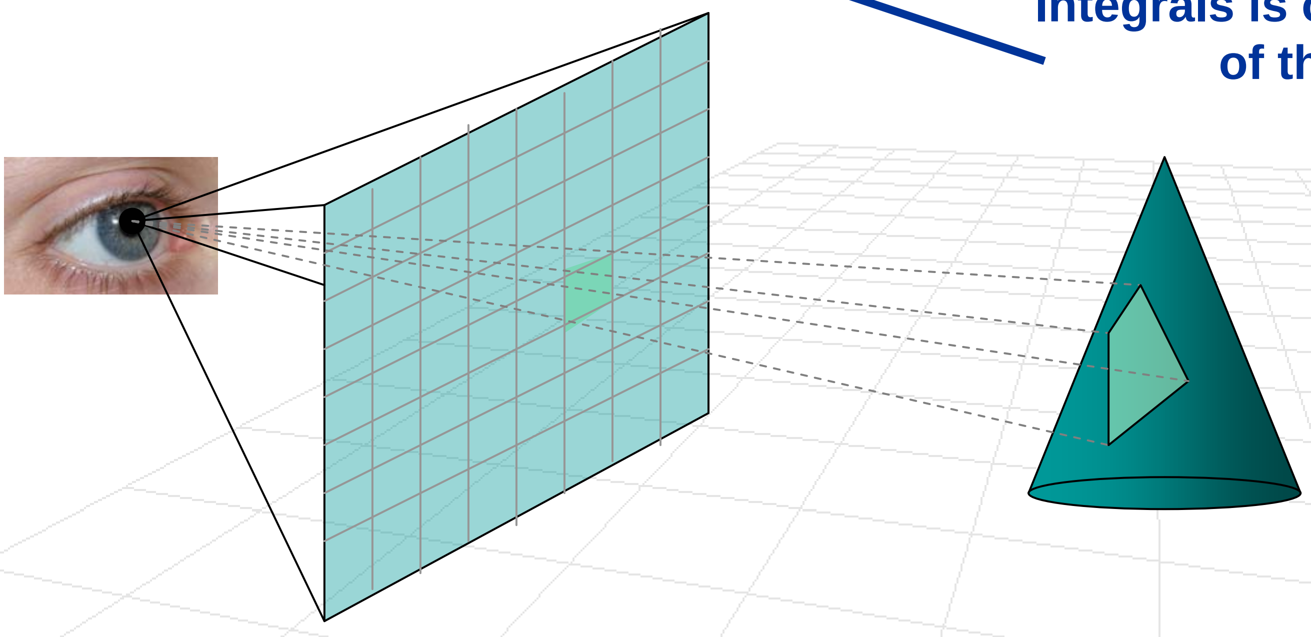


Irradiance at a Pixel

- To compute the color of the pixel, we need to compute **total light energy** (flux) **passing through the pixel** (rectangle) (i.e. we need to compute the total irradiance at a pixel)

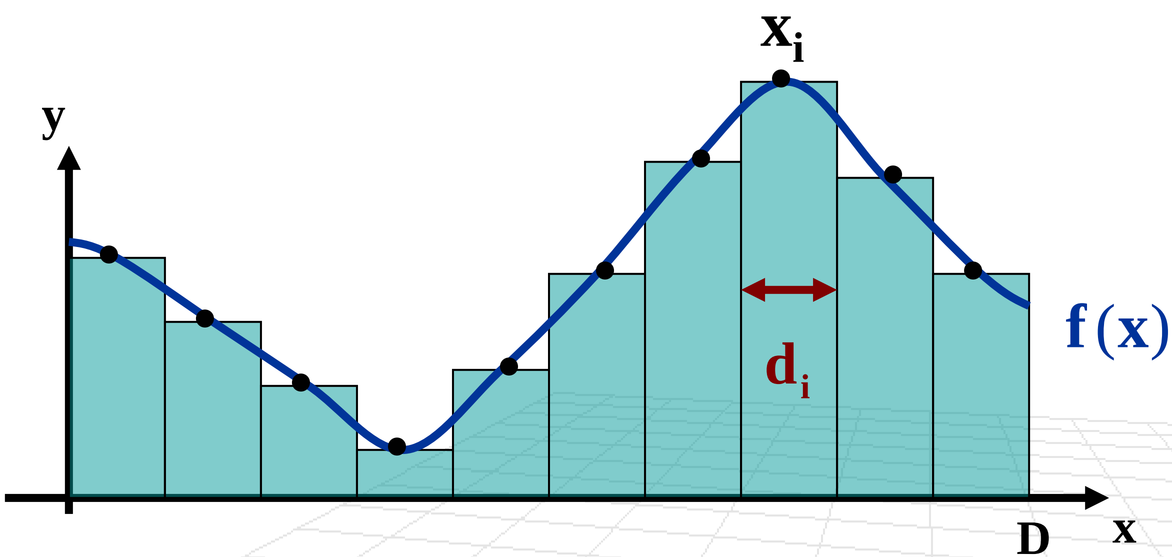
$$\Phi_{i,j} = \int_{\alpha_{min} \le \alpha \le \alpha_{max}} \int_{\beta_{min} \le \beta \le \beta_{max}} \mathbf{H}(\alpha, \beta)\, d\alpha\, d\beta$$

Integrals is over the extent of the pixel



Numerical Integration (1D Case)

- **Remember:** integral is an area under the curve
- We can approximate any integral numerically as follows



$$\sum_{i=1}^{N} d_i f(x_i) \xrightarrow[N \to \infty]{} \int_0^D f(x)\,dx$$

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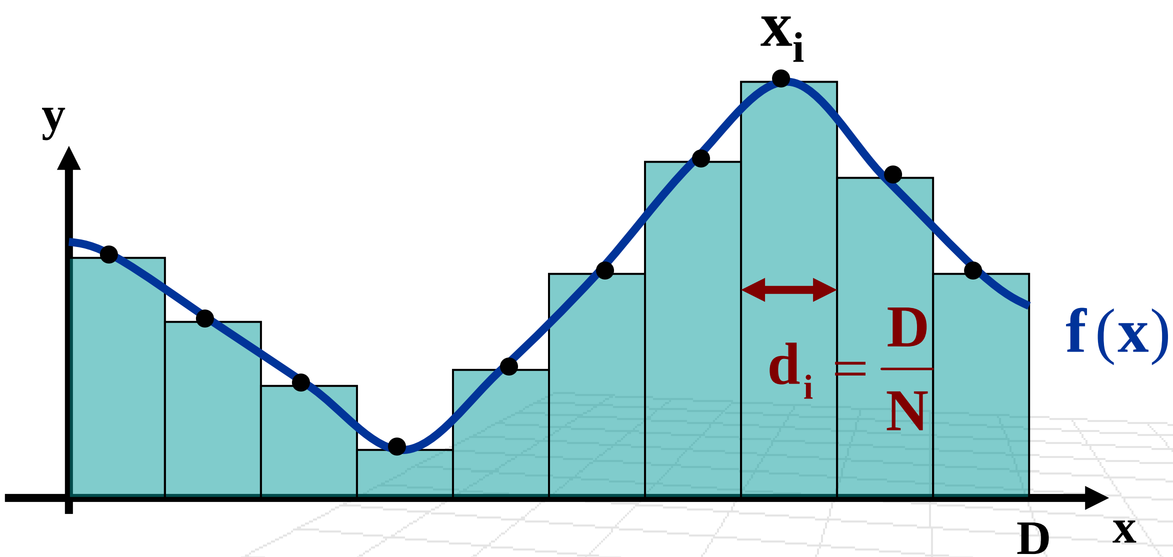
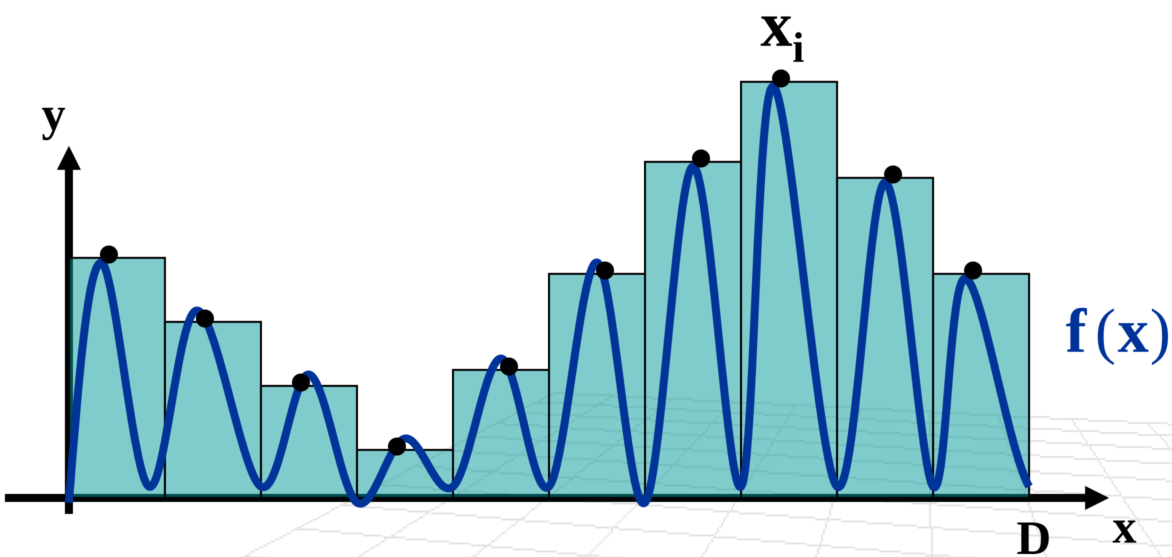


$$\int_0^D f(x)\,dx \approx \sum_{i=1}^{N} \frac{D}{N} f(x_i)$$

Numerical Integration (1D Case)

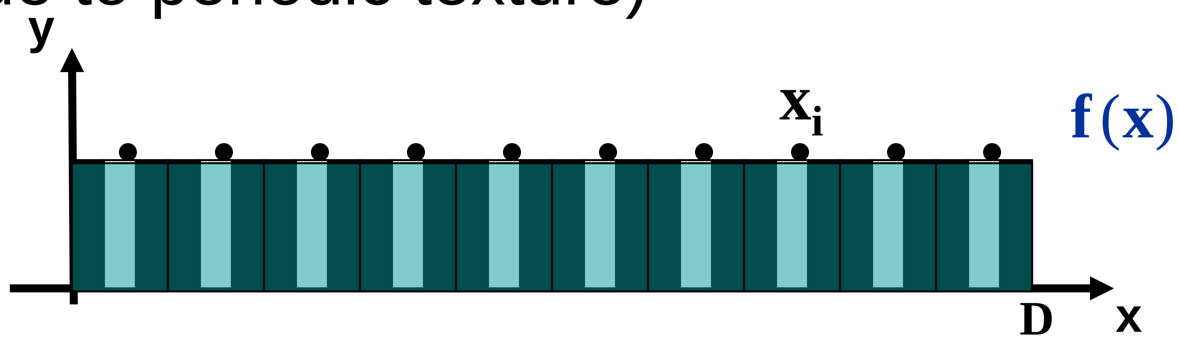
- **Problem:** what if we are really unlucky and our signal has the same structure as sampling?



$$\int_0^D f(x)\,dx \approx \sum_{i=1}^{N} \frac{D}{N} f(x_i)$$

Monte Carlo Integration

- **Idea:** randomize points $\mathbf{x_i}$ to avoid structured noise (e.g. due to periodic texture)

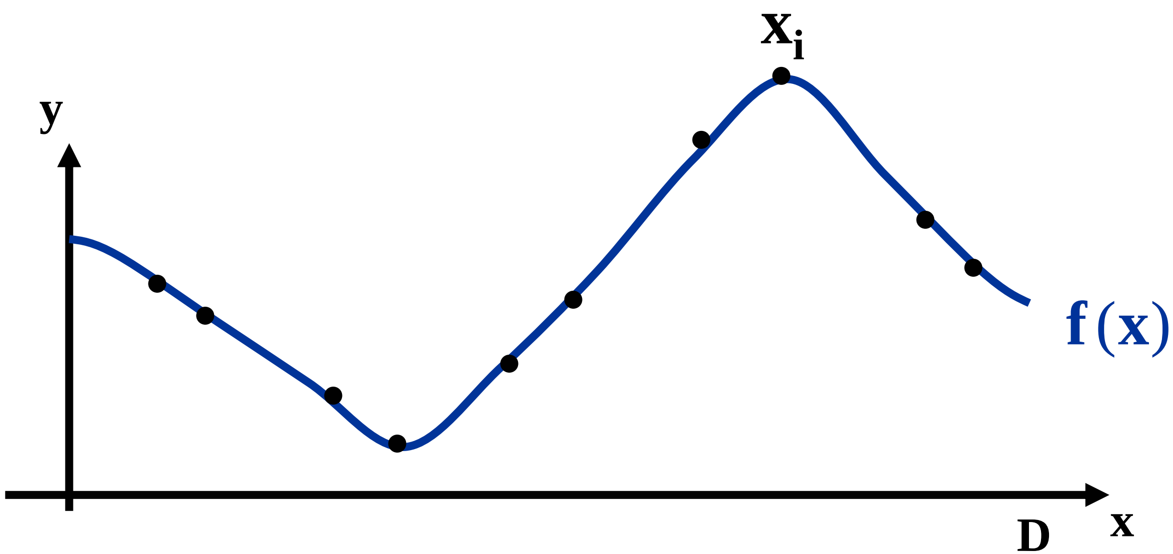


- Draw **N** random samples $\mathbf{x_i}$ independently from uniform distribution **Q(x)=U[0,D]** (i.e. **Q(x) = 1/D** is the uniform probability density function)
- Then approximation to the integral becomes

$$\frac{1}{N}\sum \mathbf{w_i f(x_i)} \approx \int \mathbf{f(x)\,dx} \quad \text{, for } \mathbf{w_i} = \frac{1}{\mathbf{Q(x_i)}}$$

- **We can also use other Q's for efficiency !!!** (a.k.a. importance sampling)

Monte Carlo Integration



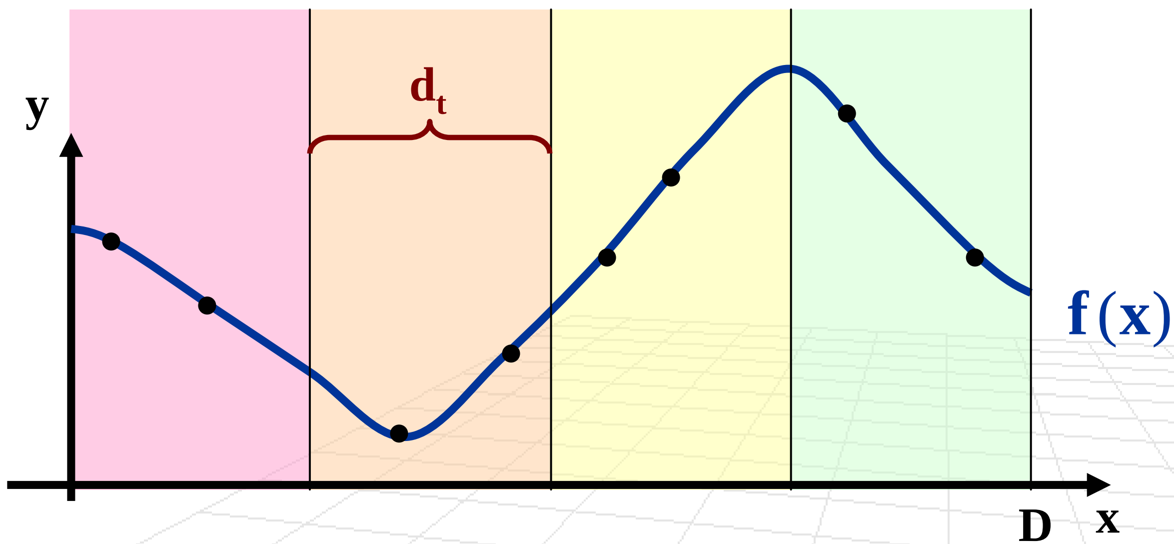
- Then approximation to the integral becomes

$$\frac{1}{N}\sum w_i f(x_i) \approx \int f(x)\,dx \quad \text{, for } w_i = \frac{1}{Q(x_i)}$$

- **We can also use other Q's for efficiency !!!** (a.k.a. importance sampling)

Stratified Sampling

- **Idea:** combination of uniform sampling plus random jitter
- Break domain into $\mathbf{T}$ intervals of widths $\mathbf{d_t}$ and $\mathbf{N_t}$ samples in interval $\mathbf{t}$



- Integral approximated using the following:

$$\sum_{t=1}^{T} \frac{1}{N_t} \sum_{j=1}^{N_t} d_t f(x_{t,j})$$