Announcements

- **Assignment 3**
  - **Programming** will be given out **first**
  - **Theory** will be given out **later**
  - Due dates will be shifted accordingly

- **Office Hours**
  - After class **today from 11-11:45**

Radiometry

Computer Graphics, CSCD18

Fall 2008

Instructor: Leonid Sigal

Radiometry

- Previously we treated light and material reflectance heuristically
  - Not physically plausible (e.g. no accounting for conservation of energy)
- To move to more advanced rendering techniques, it is necessary to treat light and reflectance more rigorously
- This involves physics and some more advance geometry

Basic Assumptions and Setup

- Basic assumptions
  - Light travels along straight lines
  - There are no delays due to the light travel through space
  - Light is scattered not absorbed (i.e. is conserved)

- With these assumptions we only need to concentrate on the geometry of lighting

Basic light related quantities

- Light **energy** is measured in **Joules**

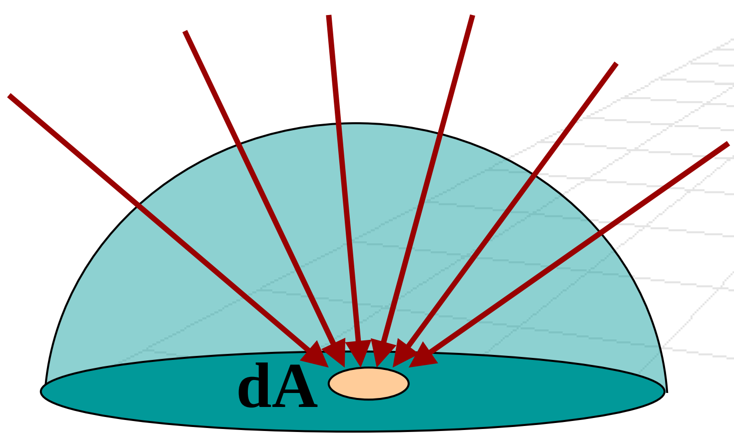
- **Power** (flux) is measured in **Watts = Joules / seconds**
  - Rate at which light energy is emitted

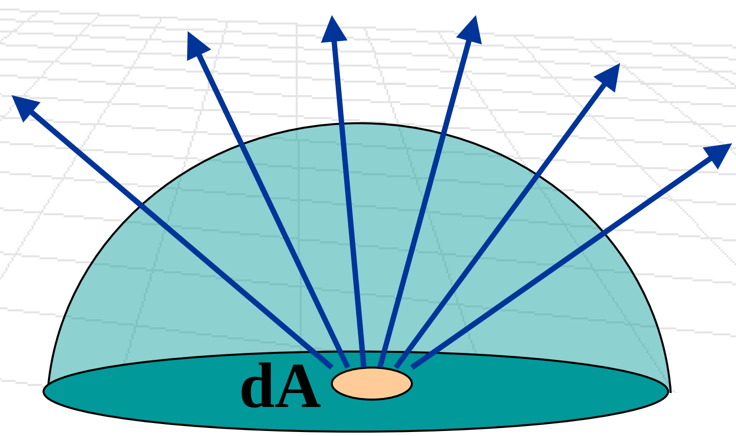
    e.g. 100 Watt bulb = 100 J/sec

  - In general, power is a function of the wavelength, but we’ll ignore that

Light

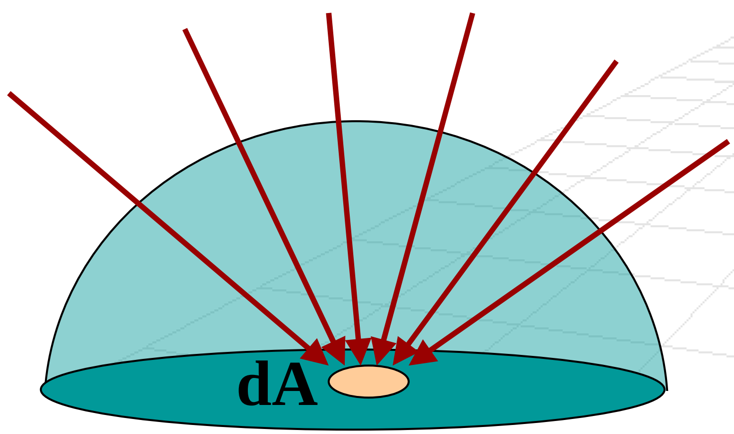
- Light is manifested as photons
  - Number of photons at a point is zero
  - Hence, we going to talk about flux density (*i.e.* number of photons per unit area)
- **Irradiance** – amount of the light falling on the surface patch (measured in Watts/meters^2)
- **Radiance** – amount of light leaving the point per area (measured in Watts/(sr * meters^2))

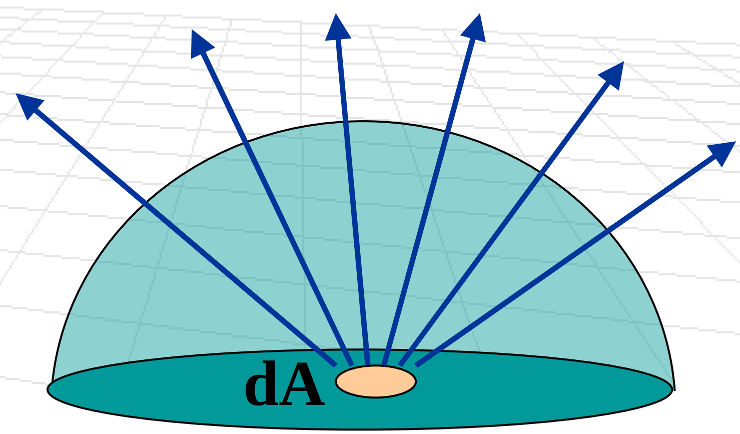




Light

- What is **steradian**?
  - Describes two-dimensional angular span (just like radians measure angular span in a plane)
  - Measure of the solid angle
- **Irradiance** – amount of the light falling on the surface patch (measured in Watts/meters^2)
- **Radiance** – amount of light leaving the point per area (measured in Watts/(**sr** * meters^2))



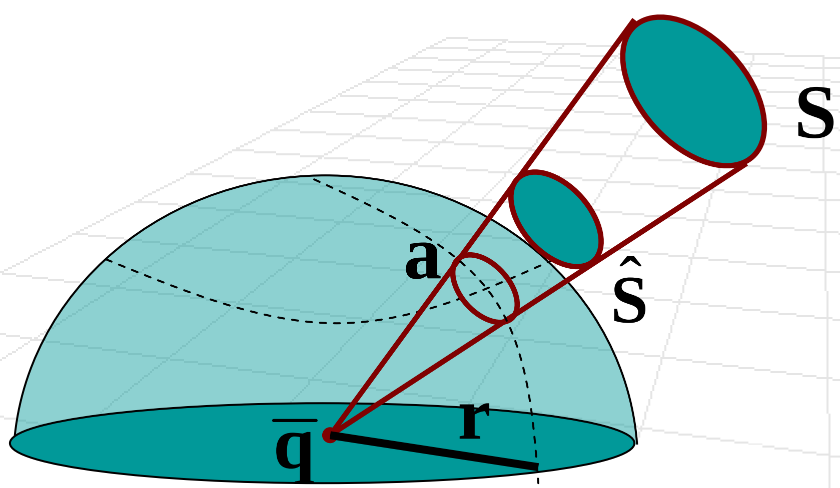


Solid Angle

- **Solid Angle** - measured as the area $\mathbf{a}$ of a patch of a sphere, divided by the squared radius $\mathbf{r}$ of the sphere

$$\omega = \frac{\mathbf{a}}{\mathbf{r}^2}$$

- **Intuition:** imagine you are at point $\bar{\mathbf{q}}$ and you look out in all possible directions, solid angle measures the amount of your view that a patch of the surface $\mathbf{S}$ is taken up



Irradiance

- What is irradiance at surface patch $\mathbf{S}$ at point $\overline{\mathbf{p}}$ due to point light source at $\overline{\mathbf{e}}$ in direction $\vec{\mathbf{d}}$, with radiance $\mathbf{I}$?
- First compute the solid angle of $\mathbf{S}$ with respect to $\overline{\mathbf{e}}$

$$\mathbf{d}\omega = \frac{\mathbf{dA_s}}{\left\|\overline{\mathbf{p}} - \overline{\mathbf{e}}\right\|^2} \boxed{(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}})}$$

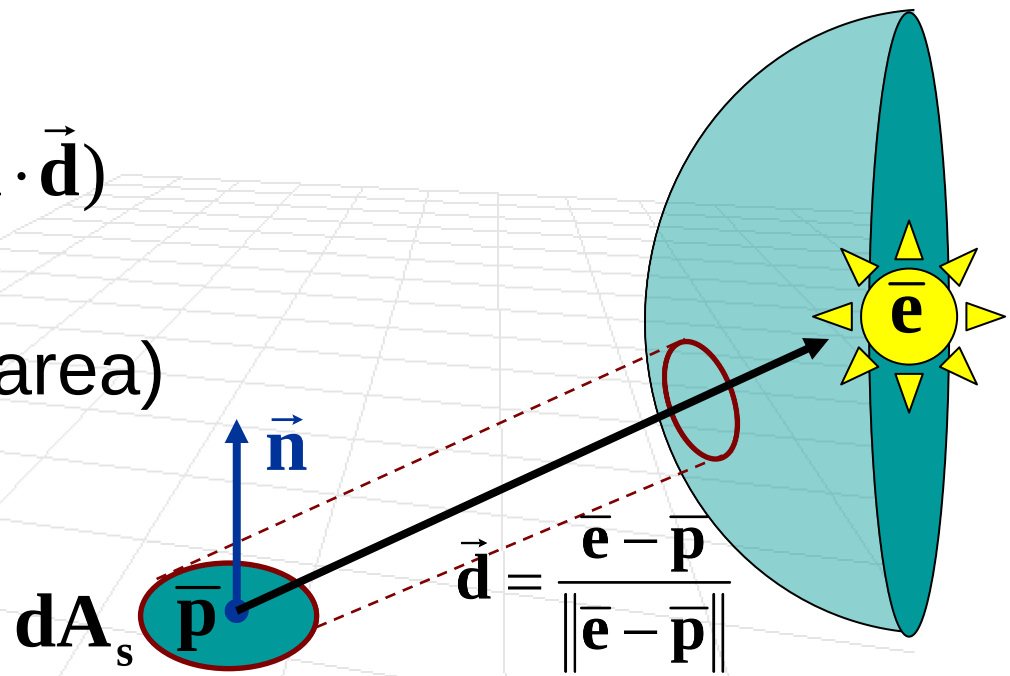
foreshortening

- Light reaching $\mathbf{S}$

$$\mathbf{S} = \mathbf{I}\,\mathbf{d}\omega = \mathbf{I}\frac{\mathbf{dA_s}}{\left\|\overline{\mathbf{p}} - \overline{\mathbf{e}}\right\|^2}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}})$$

- Irradiance (divide by area)

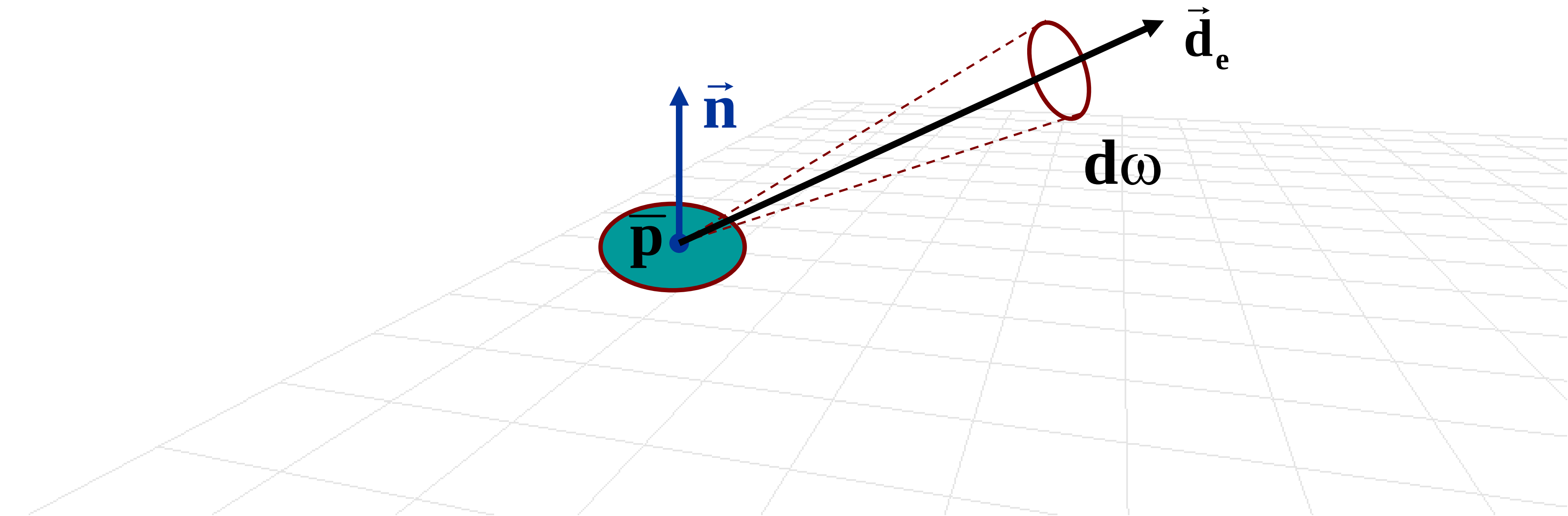
$$\mathbf{H}(\overline{\mathbf{p}}) = \frac{\mathbf{I}\,\mathbf{d}\omega}{\mathbf{dA_s}} = \frac{\mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}})}{\left\|\overline{\mathbf{p}} - \overline{\mathbf{e}}\right\|^2}$$



Radiance

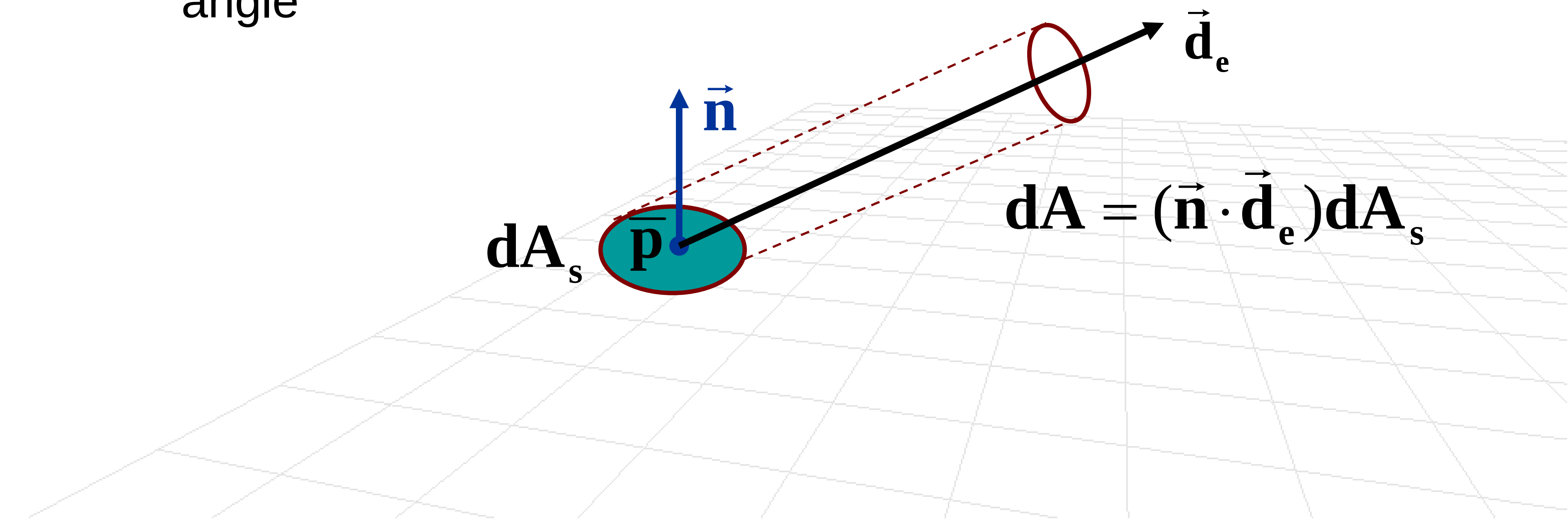
- Light emitted in direction $\vec{\mathbf{d}}_{\mathbf{e}}$ through small surface patch $\mathbf{S}$ at point $\overline{\mathbf{p}}$, is called radiance $\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{d}})$

- We need to integrate this quantity over all possible directions to obtain the **radiosity** (or **radiant exitance**)



Radiance

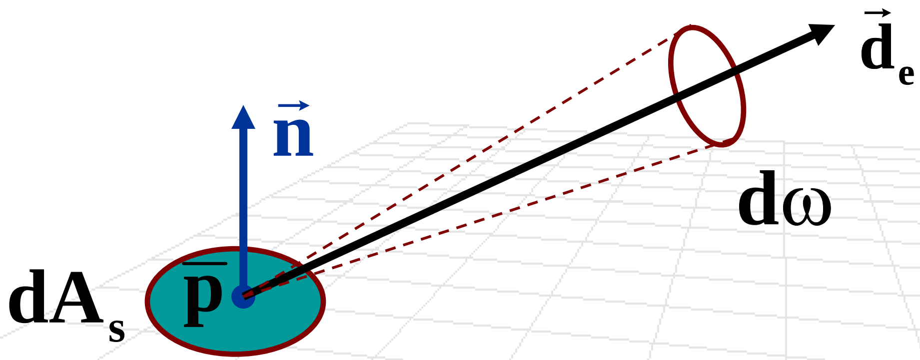
- Light emitted in direction $\vec{\mathbf{d}}_{\mathbf{e}}$ through small surface patch $\mathbf{S}$ at point $\overline{\mathbf{p}}$, is called radiance $\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{d}})$
- We need to integrate this quantity over all possible directions to obtain the **radiosity** (or **radiant exitance**)
  - But we need to account for foreshortened surface are per solid angle



Radiance

- Light emitted in direction $\vec{\mathbf{d}}_\mathbf{e}$ through small surface patch $\mathbf{S}$ at point $\overline{\mathbf{p}}$, is called radiance $\mathbf{L}(\overline{\mathbf{p}},\vec{\mathbf{d}})$

- We need to integrate this quantity over all possible directions to obtain the **radiosity** (or **radiant exitance**)
  - But we need to account for foreshortened surface are per solid angle

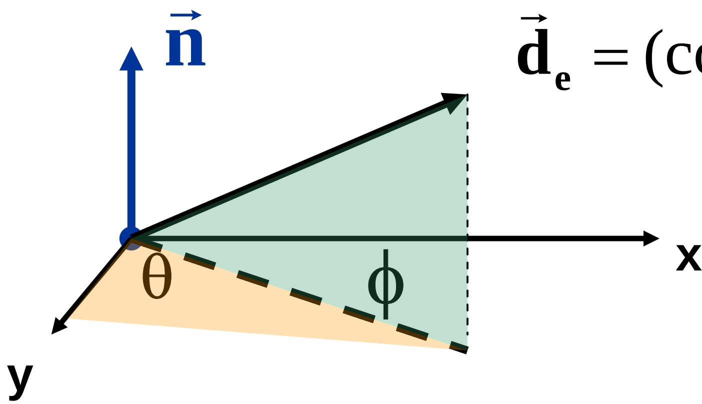


- Taking this into account we get

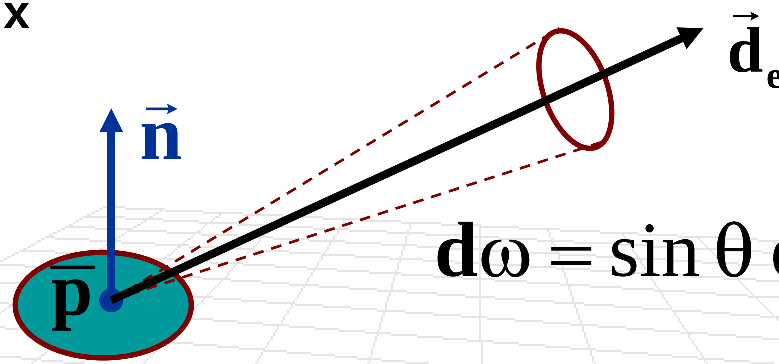
$$\mathbf{E}(\overline{\mathbf{p}}) = \int_{\vec{\mathbf{d}}_\mathbf{e} \in \Omega_\mathbf{e}} \mathbf{L}(\overline{\mathbf{p}},\vec{\mathbf{d}}_\mathbf{e})(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_\mathbf{e})\,\mathbf{d}\omega$$

Radiance

- In spherical coordinates, we can express this as a double integral (assuming infinitesimally small patch)



$$\vec{d}_e = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta)$$



$$d\omega = \sin\theta \, d\theta \, d\phi$$

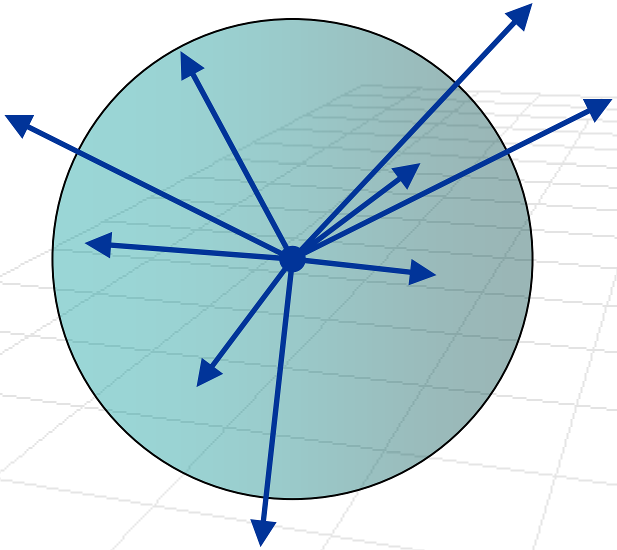
$$E(\bar{p}) = \int_{\vec{d}_e \in \Omega_e} L(\bar{p}, \vec{d}_e)(\vec{n} \cdot \vec{d}_e) \, d\omega$$

$$E(\bar{p}) = \int_\phi \int_\theta L(\bar{p}, \vec{d}_e)(\vec{n} \cdot \vec{d}_e) \sin\theta \, d\theta \, d\phi$$

Irradiance from Radiance

- We can get irradiance by integrating radiance over the entire sphere
- **Intuition:** Light that is hitting the surface is equal to the light emitted by everything else in the direction of the point

$$\mathbf{H}(\overline{\mathbf{p}}) = \int_{\phi}\int_{\theta} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i) \sin\theta \, \mathbf{d}\theta \, \mathbf{d}\phi$$



Radiance vs. Irradiance

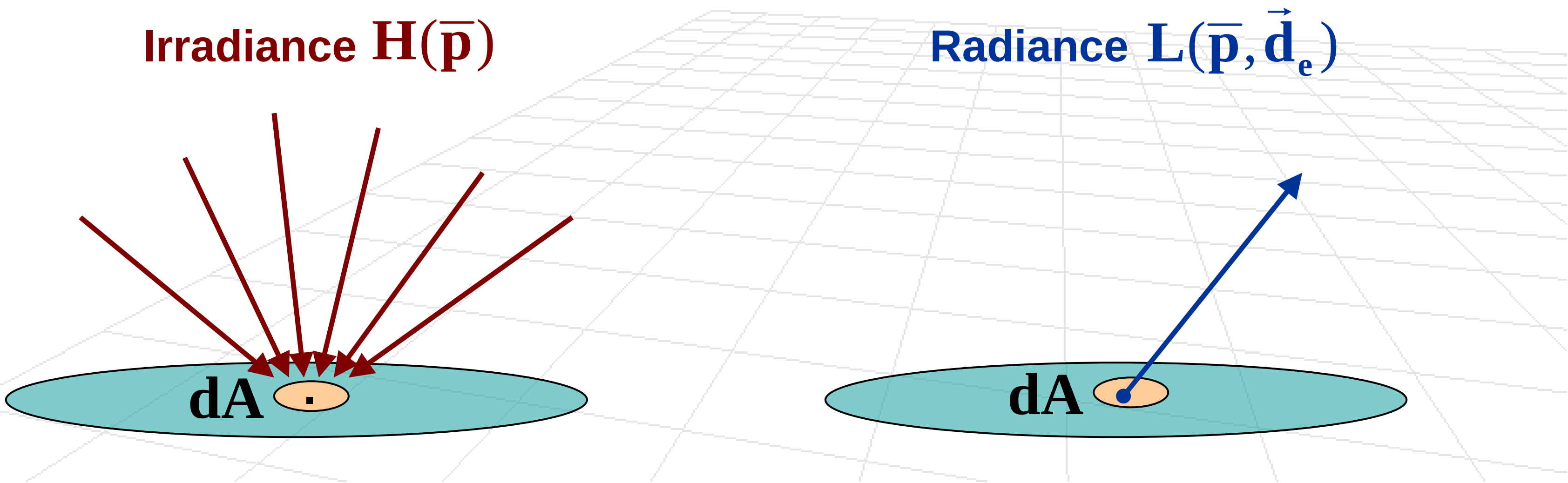
- **Radiance**
  - Describes light emitted from a surface (per area)
  - Function of direction
  - Units: $\mathbf{W} \cdot \mathbf{sr}^{-1} \cdot \mathbf{m}^{-2}$
- **Irradiance**
  - Describes light incident on a surface
  - Not a directional quantity
  - Units: $\mathbf{W} \cdot \mathbf{m}^{-2}$
- From the radiance emitted from one surface we can compute the incidence irradiance at a nearby surface

Bidirectional Reflectance Distribution Function (BRDF)

- **BRDF:** Ratio of emittant to incident light (i.e. radiance to irradiance)

$$\rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i) = \frac{\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e)}{\mathbf{H}(\overline{\mathbf{p}})}$$

Intuition: what fraction of the light entering along one direction willbe emitted in the other

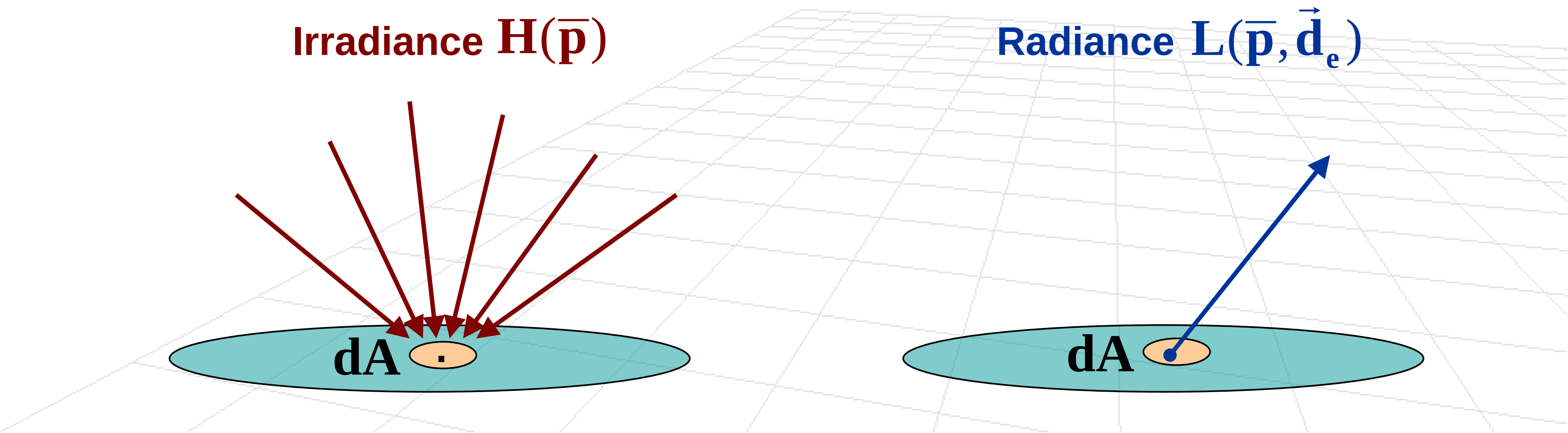


Bidirectional Reflectance Distribution Function (BRDF)

- **BRDF:** Ratio of emittant to incident light (i.e. radiance to irradiance)

$$\rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i) = \frac{\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e)}{\mathbf{H}(\overline{\mathbf{p}})}$$

- Models reflectance of simple materials
- Often BRDF must be empirically determined (measured in a laboratory)

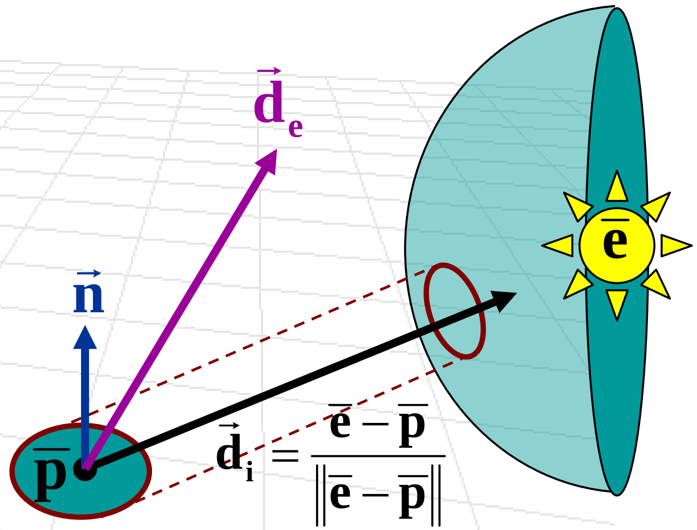


Point Light Sources

- Let's compute surface radiance for a point light source with radiant intensity **I**
  - I = flux for a solid angle **dw**
- We already know (from earlier slides) that for a point light source irradiance is given by: $\mathbf{H}(\bar{\mathbf{p}}) = \frac{\mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i)}{\|\bar{\mathbf{p}} - \bar{\mathbf{e}}\|^2}$
- We can then get surface radiance by rearranging terms in the definition of BRDF

$$\mathbf{L}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_e) = \rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i)\mathbf{H}(\bar{\mathbf{p}})$$

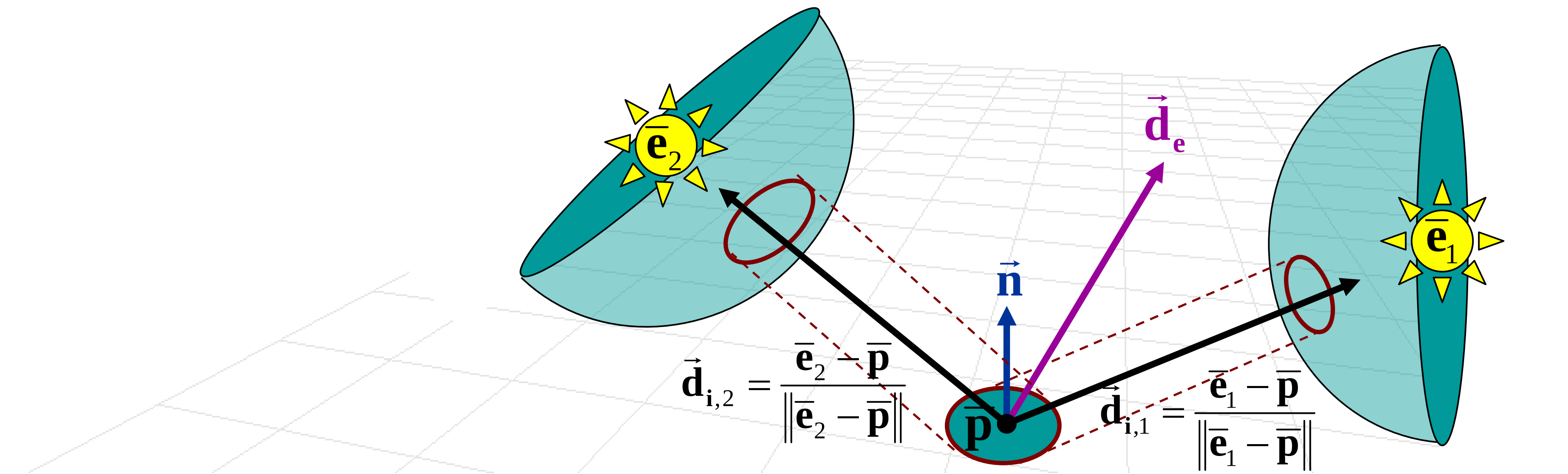
$$= \rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i)\frac{\mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i)}{\|\bar{\mathbf{p}} - \bar{\mathbf{e}}\|^2}$$



Multiple Point Light Sources

- Simple to handle, since light is additive

$$\mathbf{L}(\overline{\mathbf{p}},\vec{\mathbf{d}}_{\mathbf{e}}) = \sum_{j=1}^{J} \rho(\vec{\mathbf{d}}_{\mathbf{e}},\vec{\mathbf{d}}_{\mathbf{i},\mathbf{j}}) \frac{\mathbf{I}(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_{\mathbf{i},\mathbf{j}})}{\left\|\overline{\mathbf{p}}-\overline{\mathbf{e}}_{\mathbf{j}}\right\|^{2}}$$



Extended Light Sources

- We can use radiance to compute required irradiance at a point by integrating over the incident directions

- Remember

$$\mathbf{H}(\overline{\mathbf{p}}) = \int_{\phi}\int_{\theta} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i) \sin\theta \, \mathbf{d}\theta \, \mathbf{d}\phi$$

hence

$$\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_e) = \int_{\phi}\int_{\theta} \rho(\vec{\mathbf{d}}_e, \vec{\mathbf{d}}_i) \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_i) \sin\theta \, \mathbf{d}\theta \, \mathbf{d}\phi$$

How much of this light is reflected in a given direction

How much light is hitting surface point

Idealizing Lighting and Reflectance

- We will consider a few special cases of the general BRDF models that facilitate lighting

- How do we do Phong lighting in terms of BRDFs?

Diffuse Reflection

- The only factor that determines appearance (radiance) of a **Lambertian** surface is irradiance (incident light)
- In other words, BRDF is constant and independent of incident and emittent direction. i.e. $\rho(\vec{\mathbf{d}}_{\mathbf{e}}, \vec{\mathbf{d}}_{\mathbf{i}}) = \rho_0$
- The radiance

$$\mathbf{L}_{\mathbf{d}}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_{\mathbf{e}}) = \rho_0 \int_{\vec{\mathbf{d}}_{\mathbf{i}} \in \Omega_{\mathbf{i}}} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_{\mathbf{i}})(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_{\mathbf{i}}) \mathbf{d}\omega_{\mathbf{i}}$$

$$\mathbf{L}_{\mathbf{d}}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_{\mathbf{e}}) = \rho_0 \int_{\vec{\mathbf{d}}_{\mathbf{i}} \in \Omega_{\mathbf{i}}} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_{\mathbf{i}}) \cos\theta_{\mathbf{i}} \mathbf{d}\omega_{\mathbf{i}}$$

- Since total irradiance must equal radiant exitance (conservation of energy), we can show that $\rho_0 = \frac{1}{\pi}$

Small proof

$$\int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)\cos\theta_i \mathbf{d}\omega_i = \int_{\vec{\mathbf{d}}_e \in \Omega_e} \rho_0 \int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)\cos\theta_i \mathbf{d}\omega_i \cos\theta_e \mathbf{d}\omega_e$$

$$\int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)\cos\theta_i \mathbf{d}\omega_i = \pi\rho_0 \int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\overline{\mathbf{p}}, -\vec{\mathbf{d}}_i)\cos\theta_i \mathbf{d}\omega_i$$

$$1 = \pi\rho_0$$

$$\rho_0 = \frac{1}{\pi}$$

Diffuse Reflection

- Despise simple BRDF, it's still hard to compute radiance because of the integral

$$\mathbf{L_d}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \int_{\vec{\mathbf{d}}_\mathbf{i} \in \Omega_\mathbf{i}} \mathbf{L}(\bar{\mathbf{p}}, -\vec{\mathbf{d}}_\mathbf{i})(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i}) d\omega_\mathbf{i}$$

- Assuming point light source helps
  - Lets assume single point light source with intensity $\mathbf{I}$
  - Then irradiance is as before $\mathbf{H}(\bar{\mathbf{p}}) = \dfrac{\mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})}{\|\bar{\mathbf{p}} - \bar{\mathbf{e}}\|^2}$

$$\mathbf{L_d}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \frac{\mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})}{\|\bar{\mathbf{p}} - \bar{\mathbf{e}}\|^2}$$

- Assuming that light is far away removes the denominator $\mathbf{L_d}(\bar{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})$ **Why?**

Diffuse Reflection

- Despise simple BRDF, it's still hard to compute radiance because of the integral

$$\mathbf{L_d}(\overline{\mathbf{p}},\vec{\mathbf{d}}_e) = \rho_0 \int_{\vec{\mathbf{d}}_i \in \Omega_i} \mathbf{L}(\overline{\mathbf{p}},-\vec{\mathbf{d}}_i)(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_i)d\omega_i$$

- Assuming point light source helps
  - Lets assume single point light source with intensity $\mathbf{I}$
  - Then irradiance is as before $\mathbf{H}(\overline{\mathbf{p}}) = \dfrac{\mathbf{I}(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_i)}{\left\|\overline{\mathbf{p}}-\overline{\mathbf{e}}\right\|^2}$

$$\mathbf{L_d}(\overline{\mathbf{p}},\vec{\mathbf{d}}_e) = \rho_0 \frac{\mathbf{I}(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_i)}{\left\|\overline{\mathbf{p}}-\overline{\mathbf{e}}\right\|^2}$$

Quantity is a constant for a surface

- Assuming that light is far away removes the denominator

$$\mathbf{L_d}(\overline{\mathbf{p}},\vec{\mathbf{d}}_e) = \rho_0\mathbf{I}(\vec{\mathbf{n}}\cdot\vec{\mathbf{d}}_i)$$

Why?

Remember the Phong model?

- Remember Phong lighting equation?

$$\mathbf{L}(\overline{\mathbf{p}}, \vec{\mathbf{c}}) = \mathbf{r_d I_d} \max(0, \vec{\mathbf{d}}_\mathbf{i} \cdot \vec{\mathbf{n}}) + \mathbf{r_a I_a} + \mathbf{r_s I_s} \max(0, \vec{\mathbf{r}} \cdot \vec{\mathbf{c}})^\alpha$$

$$\mathbf{r_d} = \rho_0 \overset{?}{\leq} \frac{1}{\pi}$$

- Assuming that light is far away removes the denominator

$$\mathbf{L_d}(\overline{\mathbf{p}}, \vec{\mathbf{d}}_\mathbf{e}) = \rho_0 \mathbf{I}(\vec{\mathbf{n}} \cdot \vec{\mathbf{d}}_\mathbf{i})$$