## Announcements

- Assignment 1
- Programming (was due Friday)
- Last day to submit for late credit was yesterday
- Assignment 2
- Theory due next Wednesday
- Programming
- Midterm
- Today 6-7pm in AA204
- All you need is a writing utensil and an ID
(no books/notes/calculators, etc.)
- You cannot leave the exam room till 7:15pm


## Last week ...

- We started taking about lighting - I introduced a Phong model


Diffuse


Specular


Ambient

$$
\mathbf{L}(\overline{\mathbf{p}}, \overrightarrow{\mathbf{c}}, \overrightarrow{\mathbf{s}})=\mathbf{r}_{\mathbf{d}} \mathbf{I}_{\mathbf{d}} \max (0, \overrightarrow{\mathbf{s}} \cdot \overrightarrow{\mathbf{n}})+\mathbf{r}_{\mathbf{s}} \mathbf{I}_{\mathbf{s}} \max (0, \overrightarrow{\mathbf{r}} \cdot \overrightarrow{\mathbf{c}})^{\alpha}+\mathbf{r}_{\mathbf{a}} \mathbf{I}_{\mathbf{a}}
$$

## Last week ...

- We started taking about lighting - I introduced a Phong model


$$
\mathbf{L}(\overline{\mathbf{p}}, \overrightarrow{\mathbf{c}}, \overrightarrow{\mathbf{s}})=\mathbf{r}_{\mathbf{d}} \mathbf{I}_{\mathbf{d}} \max (0, \overrightarrow{\mathbf{s}} \cdot \overrightarrow{\mathbf{n}})+\mathbf{r}_{\mathbf{s}} \mathbf{I}_{\mathbf{s}} \max (0, \overrightarrow{\mathbf{r}} \cdot \overrightarrow{\mathbf{c}})^{\alpha}+\mathbf{r}_{\mathbf{a}} \mathbf{I}_{\mathrm{a}}
$$

## Shading

Computer Graphics, CSCD18
Fall 2008
Instructor: Leonid Sigal

## Shading

- Goal: use light/reflectance model we derived last week to shade/color facets of polygonal mesh
- We know how to color a point on objects surface given a point, a normal at that point, a light source, and a camera position (Phong model)
- But, geometry is not modeled using points (too expensive), it is modeled using polygonal meshes.
- This is why we need shading


## Basic setup

## Assume we know

$\overline{\mathbf{e}}^{\mathbf{w}} \quad$ - center of projection (position of eye) in world coordinates

- position of the point light source in world coordinates
$\mathbf{I}_{\mathrm{a}}, \mathbf{I}_{\mathrm{d}}, \mathbf{I}_{\mathrm{s}}$ - intensity of ambient, diffuse, and secular light sources
$\mathbf{r}_{\mathbf{a}}, \mathbf{r}_{\mathbf{d}}, \mathbf{r}_{\mathbf{s}}$ - reflection coefficients of ambient, diffuse, and secular light sources
$\alpha \quad-$ exponent controlling width of the specular highlight


## How do we shade a triangle?

## Flat Shading

- Idea: Fill each triangle with single color
- Let us assume we have a triangle with vertices $\overline{\mathbf{p}}_{1}, \overline{\mathbf{p}}_{2}, \overline{\mathbf{p}}_{3}$ in CCW order
- We can then compute the normal, $\overrightarrow{\mathbf{n}}=\frac{\left(\overline{\mathbf{p}}_{2}-\overline{\mathbf{p}}_{\mathbf{1}}\right) \times\left(\overline{\mathbf{p}}_{3}-\overline{\mathbf{p}}_{1}\right)}{\left\|\left(\overline{\mathbf{p}}_{2}-\overline{\mathbf{p}}_{1}\right) \times\left(\overline{\mathbf{p}}_{3}-\overline{\mathbf{p}}_{1}\right)\right\|}$
- And shade entire triangle using Phong model

$$
\mathbf{L}\left(\overline{\mathbf{p}}, \overline{\mathbf{e}}^{\mathrm{w}}, \overline{\mathbf{I}}^{\mathbf{w}}\right)=\mathbf{r}_{\mathbf{d}} \mathbf{I}_{\mathbf{d}} \max (0, \overrightarrow{\mathbf{s}} \cdot \overrightarrow{\mathbf{n}})+\mathbf{r}_{s} \mathbf{I}_{\mathrm{s}} \max (0, \overrightarrow{\mathbf{r}} \cdot \overrightarrow{\mathbf{c}})^{\alpha}+\mathbf{r}_{\mathbf{a}} \mathbf{I}_{\mathbf{a}}
$$



## Flat Shading



Foley, van Dam, Feiner, Hughes, Plate II. 29

## Issues with Flat Shading

- For large faces secularities are impractical, since highlight is often sharp
- Because of this, typically the secular term is dropped
- Mesh binderies are visible
- People are very sensitive to this
- Solutions
- Use small patches (but this is inefficient)
- Use interpolative shading



## Interpolative Shading

- Idea: Average intensities at vertices of the triangle to smoothly interpolate over pixels within a face
- Algorithm, for a triangular face with vertices $\overline{\mathbf{p}}_{1}, \overline{\mathbf{p}}_{2}, \overline{\mathbf{p}}_{3}$
- Compute normals at each vertex
- Compute radiance $\mathbf{E}_{\mathbf{j}}=\mathbf{L}\left(\overline{\mathbf{p}}_{\mathbf{j}}, \overline{\mathbf{e}}^{\mathbf{w}}, \overline{\mathbf{l}}^{\mathbf{w}}\right)$ for each vertex point $\overline{\mathbf{p}}_{\mathbf{j}}$

$$
\mathbf{L}\left(\overline{\mathbf{p}}_{\mathbf{j}}, \overline{\mathbf{e}}^{\mathrm{w}}, \overline{\mathbf{l}}^{\mathbf{w}}\right)=\mathbf{r}_{\mathbf{d}} \mathbf{I}_{\mathbf{d}} \max \left(0, \overrightarrow{\mathbf{s}}_{\mathbf{j}} \cdot \overrightarrow{\mathbf{n}}_{\mathbf{j}}\right)+\mathbf{r}_{\mathbf{s}} \mathbf{I}_{\mathrm{s}} \max \left(0, \overrightarrow{\mathbf{r}}_{\mathbf{j}} \cdot \overrightarrow{\mathbf{c}}_{\mathbf{j}}\right)^{\alpha}+\mathbf{r}_{\mathbf{a}} \mathbf{I}_{\mathbf{a}}
$$

- Project vertices onto image plane
- Fill polygon by interpolating radiance along the triangle (scan conversion)


## Interpolative Shading in Detail

- Compute normals at each vertex
- Many approaches are possible
- Given parametric shape, compute normal $\overrightarrow{\mathbf{n}}_{\mathrm{j}}$ when sampling vertices of the mesh $\overline{\mathbf{p}}_{\mathbf{j}}$
- Implicit form

$$
\overrightarrow{\mathbf{n}}_{\mathbf{j}}\left(\overline{\mathbf{p}}_{\mathbf{j}}\right)=\nabla \mathbf{f}\left(\overline{\mathbf{p}}_{\mathbf{j}}\right)
$$

- Explicit form

$$
\overrightarrow{\mathbf{n}}_{\mathbf{j}}\left(\overline{\mathbf{p}}_{\mathbf{j}}\right)=\left.\frac{\partial \mathbf{s}(\alpha, \beta)}{\partial \alpha}\right|_{\alpha_{0}, \beta_{0}} \times\left.\frac{\partial \mathbf{s}(\alpha, \beta)}{\partial \beta}\right|_{\alpha_{0}, \beta_{0}}
$$

- Let $\overrightarrow{\mathbf{n}}_{\mathrm{j}}$ be average of "face normals" of all adjacent faces



## Interpolative Shading in Detail

- Compute radiance $\mathbf{E}_{\mathbf{j}}$ for each vertex point $\mathbf{p}_{\mathbf{j}}$
- Same as in flat shading (using Phong model)

$$
\mathbf{E}_{\mathbf{j}}=\mathbf{L}\left(\overline{\mathbf{p}}_{\mathbf{j}}, \overline{\mathbf{e}}^{\mathrm{w}}, \overline{\mathbf{l}}^{w}\right)=\mathbf{r}_{\mathbf{d}} \mathbf{I}_{\mathbf{d}} \max \left(0, \overrightarrow{\mathbf{s}}_{\mathbf{j}} \cdot \overrightarrow{\mathbf{n}}_{\mathbf{j}}\right)+\mathbf{r}_{\mathbf{s}} \mathbf{I}_{\mathrm{s}} \max \left(0, \overrightarrow{\mathbf{r}}_{\mathbf{j}} \cdot \overrightarrow{\mathbf{c}}_{\mathbf{j}}\right)^{\alpha}+\mathbf{r}_{\mathbf{a}} \mathbf{I}_{\mathbf{a}}
$$



## Interpolative Shading in Detail

- Project onto image plane with pseudodepth
- So for each vertex we have $\mathbf{x}_{\mathbf{j}}^{*}, \mathbf{y}_{\mathbf{j}}^{*}, \mathbf{d}_{\mathbf{j}}=\mathbf{z}_{\mathbf{j}}^{*}, \mathbf{E}_{\mathbf{j}}$



## Interpolative Shading in Detail

- Scan conversion with linear interpolation of both pseudodepth ( $\mathbf{d}_{\mathbf{j}}$ ) and radiance values $\left(\mathbf{E}_{\mathrm{j}}\right)$
- use z-buffer to handle visibility



## Algorithm (part 1)

- For each edge between ( $\mathbf{x}_{\mathrm{b}}^{*}, \mathbf{y}_{\mathrm{b}}^{*}, \mathbf{d}_{\mathrm{b}}, \mathbf{E}_{\mathrm{b}}$ ) and $\left(\mathbf{x}_{\mathrm{a}}^{*}, \mathbf{y}_{\mathrm{a}}^{*}, \mathbf{d}_{\mathrm{a}}, \mathbf{E}_{\mathrm{a}}\right)$ ordered such that $\mathbf{y}_{\mathrm{a}}^{*}>\mathbf{y}_{\mathrm{b}}^{*}$ $\mathrm{x}=\mathrm{x}_{\mathrm{b}}^{*}, \quad \Delta \mathrm{x}=\left(\mathrm{x}_{\mathrm{a}}^{*}-\mathrm{x}_{\mathrm{b}}^{*}\right)\left(\mathrm{y}_{\mathrm{a}}^{*}-\mathrm{y}_{\mathrm{b}}^{*}\right)$
$\mathbf{d}=\mathbf{d}_{\mathrm{b}}, \quad \Delta \mathbf{d}=\left(\mathbf{d}_{\mathrm{a}}-\mathbf{d}_{\mathrm{b}}\right) /\left(\mathbf{y}_{\mathrm{a}}^{*}-\mathbf{y}_{\mathrm{b}}^{*}\right)$
$\mathbf{E}=\mathbf{E}_{\mathrm{b}}, \quad \Delta \mathbf{E}=\left(\mathbf{E}_{\mathrm{a}}-\mathbf{E}_{\mathrm{b}}\right) /\left(\mathbf{y}_{\mathrm{a}}^{*}-\mathbf{y}_{\mathrm{b}}^{*}\right)$
$\square \operatorname{For}\left(\mathbf{y}=\mathbf{y}_{\mathbf{b}} ; \mathbf{y}<\mathbf{y}_{\mathbf{a}} ; \mathbf{y}^{++}\right)$
Place ( $\mathbf{x}, \mathbf{d}, \mathbf{E}$ ) in active edge list (AEL) at scanline $\mathbf{y}$

$$
\begin{aligned}
& \mathbf{x}=\mathbf{x}+\Delta \mathbf{x} \\
& \mathbf{d}=\mathbf{d}+\Delta \mathbf{d} \\
& \mathbf{E}=\mathbf{E}+\Delta \mathbf{E}
\end{aligned}
$$

$\mathbf{x}_{1}^{*}, \mathbf{y}_{1}^{*}, \mathbf{d}_{1}, \mathbf{E}_{1}$
$\mathbf{x}_{3}^{*}, \mathbf{y}_{3}^{*}, \mathbf{d}_{3}, \mathbf{E}_{3}$
$\mathbf{x}_{2}^{*}, \mathbf{y}_{2}^{*}, \mathbf{d}_{2}, \mathbf{E}_{2}$

Algorithm (part 2)

- For each scanline between $\min \left(\mathbf{y}_{1}, \mathbf{y}_{2}, \mathbf{y}_{3}\right)$ and $\max \left(\mathbf{y}_{1} \cdot \mathbf{y}_{2}, \mathbf{y}_{3}\right)$
- Extract ( $\mathbf{x}_{\mathrm{a}}, \mathbf{d}_{\mathbf{a}}, \mathbf{E}_{\mathrm{a}}$ ) and ( $\left.\mathbf{x}_{\mathrm{a}}, \mathbf{d}_{\mathbf{a}}, \mathbf{E}_{\mathrm{a}}\right)$ from AEL where $\mathbf{x}_{\mathrm{a}}>\mathbf{x}_{\mathrm{b}}$

$$
\begin{array}{ll}
\mathbf{d}=\mathbf{d}_{b}, & \Delta \mathbf{d}=\left(\mathbf{d}_{\mathrm{a}}-\mathbf{d}_{\mathrm{b}}\right) /\left(\mathbf{x}_{\mathrm{a}}-\mathbf{x}_{\mathrm{b}}\right) \\
\mathbf{E}=\mathbf{E}_{\mathrm{b}}, & \Delta \mathbf{E}=\left(\mathbf{E}_{\mathrm{a}}-\mathbf{E}_{\mathrm{b}}\right) /\left(\mathbf{x}_{\mathrm{a}}-\mathbf{x}_{\mathrm{b}}\right)
\end{array}
$$

$\square \operatorname{For}\left(\mathbf{x}=\mathbf{x}_{\mathbf{b}} ; \mathbf{x}<\mathbf{x}_{\mathbf{a}} ; \mathbf{x}^{++}\right)$

$$
\begin{aligned}
& \text { if }(\mathbf{d}<\text { z-buffer }(\mathbf{x}, \mathbf{y})) \\
& \quad \quad \text { putpixel }(\mathbf{x}, \mathbf{y}, \mathbf{E}) \\
& \quad \quad \text { z-buffer }(\mathbf{x}, \mathbf{y})=\mathbf{d} \\
& \text { end } \\
& \mathbf{d}=\mathbf{d}+\Delta \mathbf{d} \\
& \mathbf{E}=\mathbf{E}+\Delta \mathbf{E}
\end{aligned}
$$

$$
\begin{aligned}
& \mathbf{x}_{1}^{*}, \mathbf{y}_{1}^{*}, \mathbf{d}_{1}, \mathbf{E}_{1} \\
& \mathbf{x}_{3}^{*}, \mathbf{y}_{3}^{*}, \mathbf{d}_{3}, \mathbf{E}_{3} \\
& \mathbf{x}_{2}^{*}, \mathbf{y}_{2}^{*}, \mathbf{d}_{2}, \mathbf{E}_{2}
\end{aligned}
$$

## Interpolative Shading in Detail

- What we just described is so called Gouraud Shading
- Advantages
- Does not produce artifacts at face boundaries (i.e. better then flat shading)
- Disadvantages
- Still hard to handle secular highlights. Why?


## Gouraud Shading



Foley, van Dam, Feiner, Hughes, Plate II. 30

## Phong Shading

- Idea: Slightly modify the Gouraud shading algorithm to correctly shade every pixel (with secularities)
(Note that phong shading and phong lighting are not one and the same)
- Algorithm, for a triangular face with vertices $\mathbf{p}_{1}, \mathbf{p}_{2}, \mathbf{p}_{3}$
- Compute normals at each vertex
- For each point on a triangle that corresponds to a pixel location interpolate the normal
- Compute radiance $\mathbf{E}_{\mathrm{j}}$ for each pixel in the projected triangle that corresponds to point within the world triangle
- Project vertices onto image plane
- Why is this batter then just doing Phong lighting?


## Phong Shading



Foley, van Dam, Feiner, Hughes, Plate II. 32

## Phong Shading

- Advantages
- Produces very accurate shading with specular highlights (better then flat shading and Gouraud shading)
- Disadvantages
- It's computationally expensive (but not on current graphics hardware)

