University of Toronto at Scarborough
Department of Computer and Mathematical Sciences

CSCD18: Computer Graphics

Midterm exam
Fall 2007

Duration: 50 minutes
No aids allowed

There are 6 pages total (including this page)

Family name: ________________________________

Given names: ________________________________

Student number: ________________________________

Question	Marks
1	__________
2	__________
3	__________
4	__________
5	__________
Total	__________

1. *[8 marks]* Answer each of the following questions in one to three sentences:

   (a) *[2 marks]* We have studied two very different classes of transformations: affine transformations and perspective projection. Basic properties of these two transformations can be summarized by considering what happens to lines (or points on those lines) as they are being transformed. In terms of lines, list one property common to both affine transformation and perspective projection; also list one property that hold for one but not the other transformation.

   Answer: Both affine transformation and perspective projection preserve lines. Affine transformation also preserves parallel lines; perspective projection does not. (Alternatively: Affine transformation also preserves relative distance along the line; perspective projection does not).

   (b) *[2 marks]* Please describe, in words, what is *pseudodeph* and what it is used for?

   Answer: Pseudodeph is a normalized depth (distance along the optical axis) in the camera's viewing volume. It is primarily used in Z-buffering to correctly render objects in the scene; by rendering objects that are closest to the image plane.

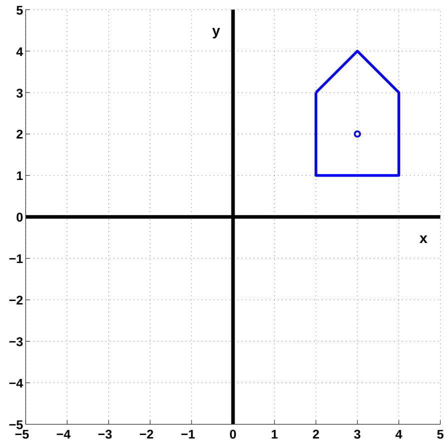
   (c) *[1 marks]* The standard pinhole camera does not contain a lens. Would the perspective projection formulation we derived for this camera in class change if we assume a presence of the thin lens at the pinhole? Why or why not?

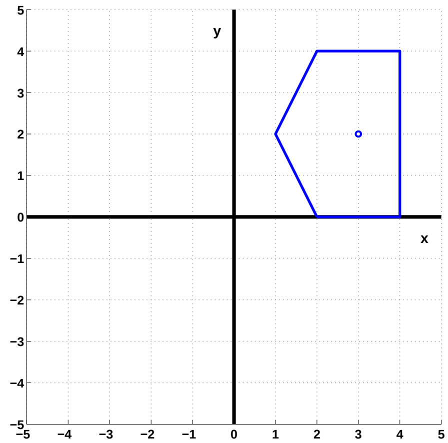
   Answer: No. Placing a thin lens at the pinhole will have no effect on where the light is being focused. Remember that rays traveling through the center of the thin lens are not deflected (they go straight through). Since our pinhole is tiny, all rays will pass though the center of the thin lens and will not be deflected.

   (d) *[3 marks]* In real cameras the lens assembly is responsible for optical zoom. This allows photographers to zoom in on distant objects without physically getting closer to them. The ideal pinhole model discussed in class has no lens assembly; but we can still simulate *zooming-in and out*. In two or three sentences, explain how?

   Answer: We can simply zoom in and out by moving our film (image plane) in or out with respect to the pinhole. Placing a film closer to the pinhole will produce a zooming-out effect; placing the film further away, will produce a zooming-in effect.

2. *[8 marks]* The diagram on the left shows a 2D *house* object, with the origin denoted by the small circle. Define a transformation matrix T, in homogeneous coordinates, that transforms the *house* on the left to a larger and rotated *house* on the right. You may define T as a composition of elementary transformation matrices.





Answer: We must apply 4 elementary transformations as follows:

(a) Translate to the origin

$$T_t = \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

(b) Scale in x-direction by 2

$$T_s = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(c) Rotate by $\frac{\pi}{2}$

$$T_r = \begin{bmatrix} cos(\frac{\pi}{2}) & -sin(\frac{\pi}{2}) & 0 \\ sin(\frac{\pi}{2}) & cos(\frac{\pi}{2}) & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

(d) Translate back

$$T_t^{-1} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

The final transformation can be written as the product of 4 transformations above:

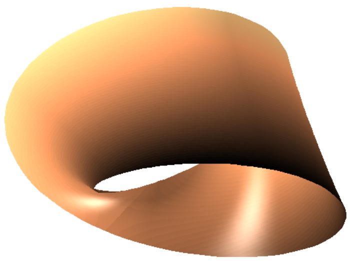
$$T = T_t^{-1} T_r T_s T_t = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 5 \\ 2 & 0 & -4 \\ 0 & 0 & 1 \end{bmatrix}.$$

Alternatively, in the above, one can switch the order of rotation and scaling, in which case the scaling must be altered to scale by 2 along the y-axis; the resulting transformation will be the same.

3. *[8 marks]* Let S be the 3D surface illustrated (called *Möbius Strip* after the German mathematicians August Ferdinand Möbius who discovered it in 1858). This surface can be defined in parametric form using the following equation:

$$S(\alpha, \beta) = \begin{bmatrix} 2cos(\alpha) + \beta cos(\alpha/2) \\ 2sin(\alpha) + \beta cos(\alpha/2) \\ \beta sin(\alpha/2) \end{bmatrix} \qquad \begin{array}{c} 0 \leq \alpha \leq 2\pi \\ -0.4 \leq \beta \leq 0.4 \end{array}$$

Find a vector that is normal to S at point $\alpha = \pi, \beta = \frac{1}{10}$. Show and explain your work.



Answer: We first must compute the vectors spanning the tangent plane by computing the partial derivatives of the surface

$$\frac{\partial S(\alpha,\beta)}{\partial \alpha} = \begin{bmatrix} -2sin(\alpha) - \frac{1}{2}\beta sin(\frac{\alpha}{2}) \\ 2cos(\alpha) - \frac{1}{2}\beta sin(\frac{\alpha}{2}) \\ \frac{1}{2}\beta cos(\frac{\alpha}{2}) \end{bmatrix}$$

$$\frac{\partial S(\alpha,\beta)}{\partial \beta} = \begin{bmatrix} cos(\frac{\alpha}{2}) \\ cos(\frac{\alpha}{2}) \\ sin(\frac{\alpha}{2}) \end{bmatrix}$$

and evaluating them at $\alpha = \pi, \beta = \frac{1}{10}$,

$$\left.\frac{\partial S(\alpha,\beta)}{\partial \alpha}\right|_{\alpha=\pi,\beta=\frac{1}{10}} = \begin{bmatrix} -2sin(\pi) - \frac{1}{2}\frac{1}{10}sin(\frac{\pi}{2}) \\ 2cos(\pi) - \frac{1}{2}\frac{1}{10}sin(\frac{\pi}{2}) \\ \frac{1}{2}\frac{1}{10}cos(\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} -\frac{1}{20} \\ -2 - \frac{1}{20} \\ 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{20} \\ -\frac{41}{20} \\ 0 \end{bmatrix}$$

$$\left.\frac{\partial S(\alpha,\beta)}{\partial \beta}\right|_{\alpha=\pi,\beta=\frac{1}{10}} = \begin{bmatrix} cos(\frac{\pi}{2}) \\ cos(\frac{\pi}{2}) \\ sin(\frac{\pi}{2}) \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

The normal of the tangent plane at the desired point is simply a surface normal at that point, hence

$$\vec{n} = \left(\left.\frac{\partial S(\alpha,\beta)}{\partial \alpha}\right|_{\alpha=\pi,\beta=\frac{1}{10}}\right) \times \left(\left.\frac{\partial S(\alpha,\beta)}{\partial \beta}\right|_{\alpha=\pi,\beta=\frac{1}{10}}\right) = \begin{bmatrix} -\frac{1}{20} \\ -\frac{41}{20} \\ 0 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.9997 \\ -0.0244 \\ 0 \end{bmatrix} \approx \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

4. *[5 marks]* Assume we have defined a camera in terms of $\bar{e}$, $\vec{u}$, $\vec{v}$, and $\vec{w}$, where $\bar{e}$ denotes the eye location (the center of projection), and the vectors $\vec{u}$, $\vec{v}$ and $\vec{w}$ form a right-handed coordinate frame (i.e., $\vec{u}$, $\vec{v}$ and $\vec{w}$ provide the directions of the camera's x, y, and z axes in the world coordinate frame). Let $\bar{p}^c$ be the representation of a point in camera-centered coordinates. Derive the homogeneous form of the transformation that maps the point $\bar{p}^c$ into its representation in world-centered coordinates, denoted $\bar{p}^w$.

   Answer: Vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ can then be used to form a change-of-basis matrix (remember linear algebra),

$$\begin{bmatrix} \vec{u} & \vec{v} & \vec{w} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \vec{u}_x & \vec{v}_x & \vec{w}_x & 0 \\ \vec{u}_y & \vec{v}_y & \vec{w}_y & 0 \\ \vec{u}_z & \vec{v}_z & \vec{w}_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

   which must be composed with a translation,

$$\begin{bmatrix} & I_{3\times 3} & & \bar{e} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & \bar{e}_x \\ 0 & 1 & 0 & \bar{e}_y \\ 0 & 0 & 1 & \bar{e}_z \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

   to yield the final camera-to-world transformation, $\bar{p}^w = M_{cw}\bar{p}^c$, where

$$M_{cw} = \begin{bmatrix} 1 & 0 & 0 & \bar{e}_x \\ 0 & 1 & 0 & \bar{e}_y \\ 0 & 0 & 1 & \bar{e}_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \vec{u}_x & \vec{v}_x & \vec{w}_x & 0 \\ \vec{u}_y & \vec{v}_y & \vec{w}_y & 0 \\ \vec{u}_z & \vec{v}_z & \vec{w}_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \vec{u}_x & \vec{v}_x & \vec{w}_x & \bar{e}_x \\ \vec{u}_y & \vec{v}_y & \vec{w}_y & \bar{e}_y \\ \vec{u}_z & \vec{v}_z & \vec{w}_z & \bar{e}_z \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

5. *[5 marks]* Assume we have a pinhole camera with an origin at $\bar{e}$ and (virtual) image plane at a focal distance f. Derive the equation of a ray, in the camera-centered coordinate frame, that maps out all visible points in space that will be projected to a single point (x^*, y^*) in the (virtual) image plane (assume far plane is at ∞).

Answer: The ray in general can be parameterized as follows:

$$\bar{r}(\lambda) = \bar{p}_0 + \lambda \vec{d}_0 \quad , \quad \lambda > 0$$

or alternatively as follows:

$$\bar{r}(\lambda) = \bar{p}_0 + |\lambda| \vec{d}_0$$

where $\bar{p}_0$ is the origin of the ray and $\vec{d}_0$ is the direction.

We know that the ray must originate at the specified point on the image plane, $\bar{p}_0 = (x^*, y^*, f)$, and shoot out in the direction of $\vec{d}_0 = (x^*, y^*, f) - \bar{e}$ (or if we want unit vector for the direction, $\frac{(x^*,y^*,f)-\bar{e}}{||(x^*,y^*,f)-\bar{e}||}$) . Since in the camera coordinate frame $\bar{e} = (0, 0, 0)$, the ray equation simply becomes:

$$\bar{r}(\lambda) = \begin{bmatrix} x^* \\ y^* \\ f \end{bmatrix} + \lambda \begin{bmatrix} x^* \\ y^* \\ f \end{bmatrix} \quad , \quad \lambda > 0.$$

Notice that this is the same as

$$\bar{r}(\lambda) = \lambda \begin{bmatrix} x^* \\ y^* \\ f \end{bmatrix} \quad , \quad \lambda > 1.$$

(There are a few alternative ways this solution be formulated.)